Application of a Non-Linear Frequency Domain Solver to the Euler and Navier-Stokes Equations

Matthew McMullen and Antony Jameson
and Juan J. Alonso
Dept. of Aeronautics & Astronautics
Stanford University

40th AIAA Aerospace Sciences Meeting
& Exhibit
Reno, NV
January 14-17, 2002

Outline

- Motivation
- Governing Equations
- Solution Method
- Gradient Based Variable Time Period
- Results - Pitching Airfoil
- Results - Cylinder
- Conclusions

Motivation

- The goal of ASCI is to calculate the unsteady flow of an aircraft gas turbine engine. This includes component simulations of the compressor, combustor, and turbine.

Component	% Wheel	Total CPU Hours
Turbine	16	2.0 million
Compressor	16	5.4 million

- The job size typically varies between 500-1,000 processors, but estimates for job length are now provided in months.

Governing Equations

- Navier-Stokes Equations in integral form

$$\int \frac{\partial W}{\partial t} dV + \oint \vec{F} \cdot \vec{N} ds = 0$$

$$W = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{bmatrix}$$

$$\vec{F}_i = f = \begin{bmatrix} \rho u \\ \rho u^2 + p - \sigma_{xx} \\ \rho u v - \sigma_{xy} \\ \rho u H - u\sigma_{xx} - v\sigma_{xy} + q_x \end{bmatrix} \qquad \vec{F}_j = g = \begin{bmatrix} \rho v \\ \rho u v - \sigma_{xy} \\ \rho v^2 + p - \sigma_{yy} \\ \rho v H - u\sigma_{xy} - v\sigma_{yy} + \end{bmatrix}$$

Governing Equations (cont.)

- Closure is provided by the following equations:

$$p = (\gamma - 1)\rho[e - \frac{1}{2}(u^2 + v^2)]$$

$$\sigma_{xx} = 2\mu u_x - \frac{2}{3}\mu(u_x + v_y)$$

$$\sigma_{yy} = 2\mu v_y - \frac{2}{3}\mu(u_x + v_y)$$

$$\sigma_{xy} = \sigma_{yx} = \mu(u_y + v_x)$$

$$q_x = \kappa\frac{\partial T}{\partial x} = -\frac{\gamma}{\gamma - 1}\frac{\mu}{Pr}\frac{\partial \frac{p}{\rho}}{\partial x}$$

$$q_y = \kappa\frac{\partial T}{\partial y} = -\frac{\gamma}{\gamma - 1}\frac{\mu}{Pr}\frac{\partial \frac{p}{\rho}}{\partial y}$$

Governing Equations (cont.)

- Approximating the volumetric integral for a given cell with volume V

$$\int \frac{\partial W}{\partial t} dV \approx V \frac{\partial W}{\partial t}$$

- Using the steady state residual variable $R(W)$ we can approximate the boundary integral as a sum of fluxes over a finite number of cell walls

$$\oint \vec{F} \cdot \vec{N} ds \approx R(W) = \sum_{j=1}^{n} \vec{F}_j \cdot \vec{S}_j$$

Governing Equations (cont.)

- Using the notation introduced above the integral form of the Navier-Stokes equation simplifies to

$$V\frac{\partial W}{\partial t} + R(W) = 0$$

- Expanding both W and $R(W)$ with a Fourier series in time

$$W = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{W}_k e^{ikt} \qquad R(W) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{R}_k e^{ikt}$$

Governing Equations (cont.)

- It follows that a periodic steady-state equation can be written for each independent wave number. We add in a pseudo-time derivative and numerically integrate the equations.

$$V\frac{d\hat{W}_k}{d\tau} + ikV\hat{W}_k + \hat{R}_k = 0$$

- Each iterative step in the solution process requires the following data flow.

Solution Method

- Since we are dealing with solving a steady system of equations, we apply established methods to accelerate the convergence.

  Multi-stage RK scheme with local time stepping
  Implicit residual averaging
  Multigrid V or W Cycle

- In addition, we are dealing with real functions where the Fourier coefficients for the positive wavenumbers are equal to the complex conjugates of the Fourier coefficients for the negative wavenumbers. This eliminates computation required to integrate the negative wave numbers forward in pseudo-time.

Gradient Based Variable Time Period (GBVTP)

- In the development of the nonlinear frequency domain method, we assume the time period of the fundamental harmonic.

- A class of problems exist where the exact frequency of the discretized equations can not be known in advance. GBVTP is an method that iteratively determines this parameter.

- The process of finding a solution to the unsteady flow equations is analogous to an optimization problem where the magnitude of the unsteady residual is minimized.

Gradient Based Variable Time Period (GBVTP)

- For this optimization problem we choose to minimize the square of the magnitude of the unsteady residual $\hat{I}_n$. Taking a derivative of this quantity we form a gradient with respect to the time period.

- This modification is not costly because the quantity $\hat{I}_n$ is already calculated while monitoring the convergence of the solution.

Gradient Based Variable Time Period (GBVTP)

- The wavenumber k is calculated by normalizing the sinusoidal period of oscillation 2π by the time period of interest T.

$$k = \frac{2\pi n}{T} \tag{1}$$

- The unsteady residual can be can then be written as a function of the time period T.

$$\hat{I}_n = \frac{i2\pi nV}{T}\hat{W}_n + \hat{R}_n \tag{2}$$

Gradient Based Variable Time Period (GBVTP)

$$\frac{1}{2}\frac{\partial\left|\hat{I}_n\right|^2}{\partial T} = \hat{I}_{nr}\frac{\partial\hat{I}_{nr}}{\partial T} + \hat{I}_{ni}\frac{\partial\hat{I}_{ni}}{\partial T} \tag{3}$$

$$\frac{\partial\hat{I}_{nr}}{\partial T} = \frac{2\pi nV\hat{W}_{ni}}{T^2} \tag{4}$$

$$\frac{\partial\hat{I}_{ni}}{\partial T} = -\frac{2\pi nV\hat{W}_{nr}}{T^2} \tag{5}$$

- The gradient can be simplified by employing cross product notation.

$$\frac{1}{2}\frac{\partial\left|\hat{I}_n\right|^2}{\partial T} = \frac{2\pi nV}{T^2}|\vec{I}_n \times \vec{W}_n| \tag{6}$$

Gradient Based Variable Time Period (GBVTP)

- The time period can be updated using the gradient information by selecting a stable step ΔT.

$$T^{n+1} = T^{n} - \Delta T \frac{\partial \left| \hat{I}_n \right|^2}{\partial T} \tag{7}$$

- Typically one can start with an initial guess in the vicinity of the final answer for the time period.

- The gradient can be used to adjust the time period at each subsequent iteration in the solution process.

Results

- Pitching Airfoil - Forced Frequency

    1. Euler
    2. Viscous
        Baldwin-Lomax Turbulence Model

- Laminar vortex shedding from a cylinder - Variable Time Period

Results - Pitching Airfoil

Description	Variable	Value
AGARD Case Number		CT6 DI 55
Airfoil		64A010
Mean Angle of Attack	α_m	0.00
Angle of Attack Variation	α_0	$\pm 1.01^\circ$
Reynolds Number	Re_∞	$12.56x10^6$
Mach Number	M_∞	0.796
Reduced Frequency	k_c	0.202

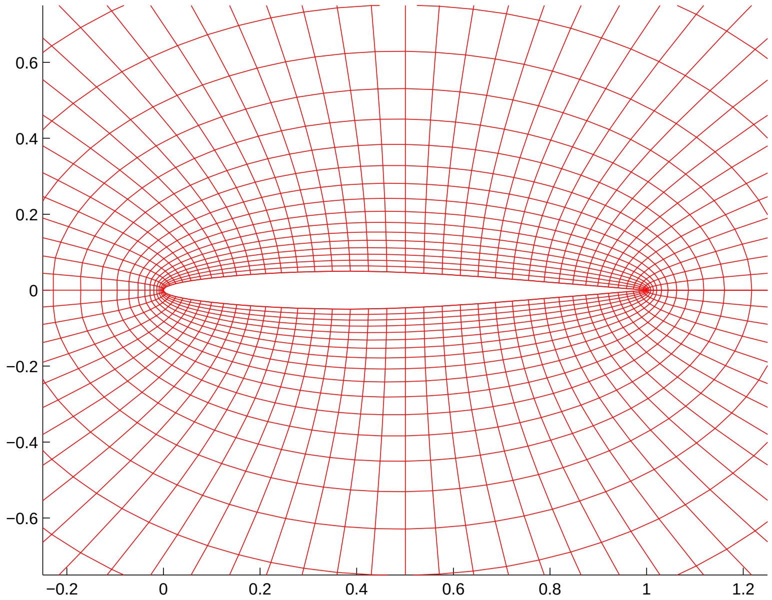
Table 1: Experiment Parameters - AGARD Report 702 by Sanford Davis

Results - Pitching Airfoil - Euler

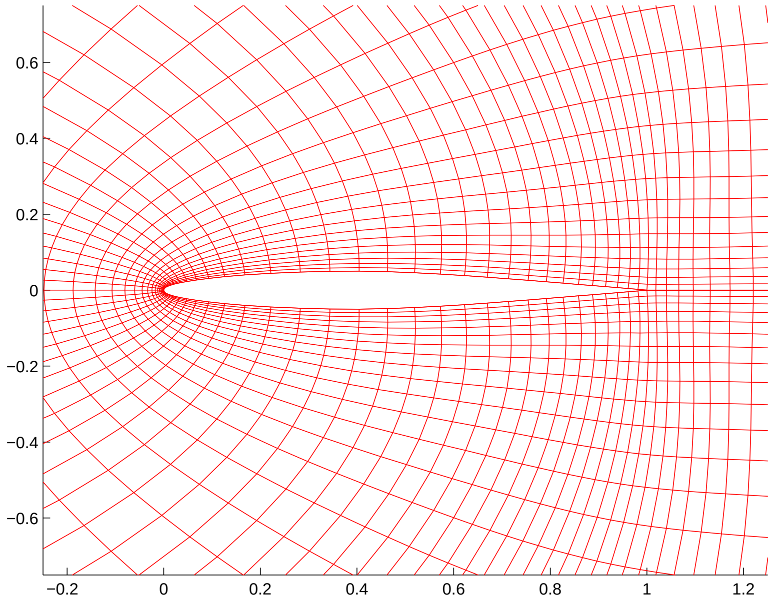
Topology	Dimensions	Mean Boundary Distance (Chords)	Mean Grid Spacing at Wall (Chords)
O-mesh	81x33	128	0.0096
O-mesh	161x33	128	0.0097
C-mesh	129x33	22	0.0087
C-mesh	193x49	22	0.0087

Table 2: Euler Grid Configurations

Results - Pitching Airfoil - Euler



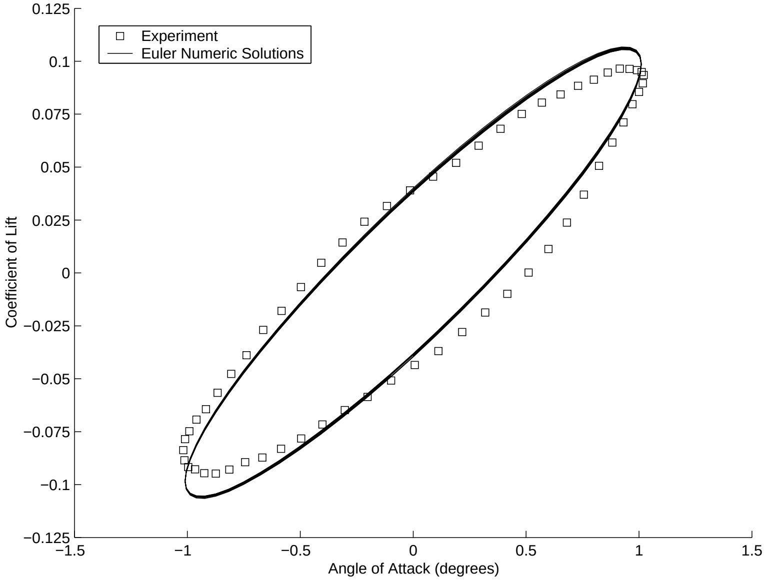
81x33 O-mesh
Conformal Mapping



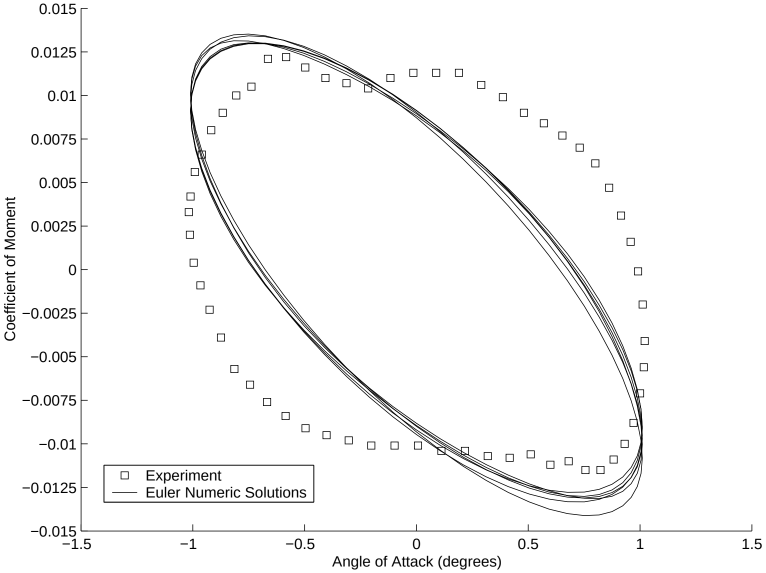
128x33 C-mesh
Hyperbolic Mesh Generation

Results - Pitching Airfoil - Euler - Cl Cm

- For each grid we ran separate cases employing 1,2 and 3 temporal modes.



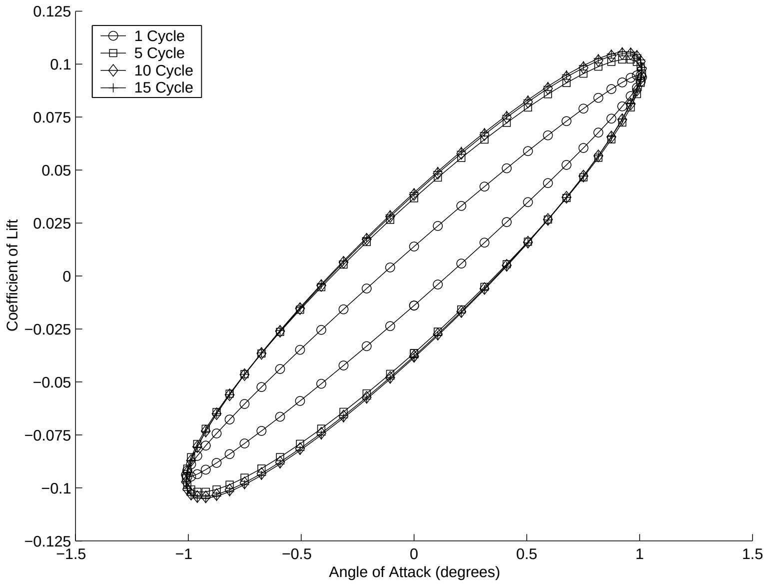
All grid and mode permutations

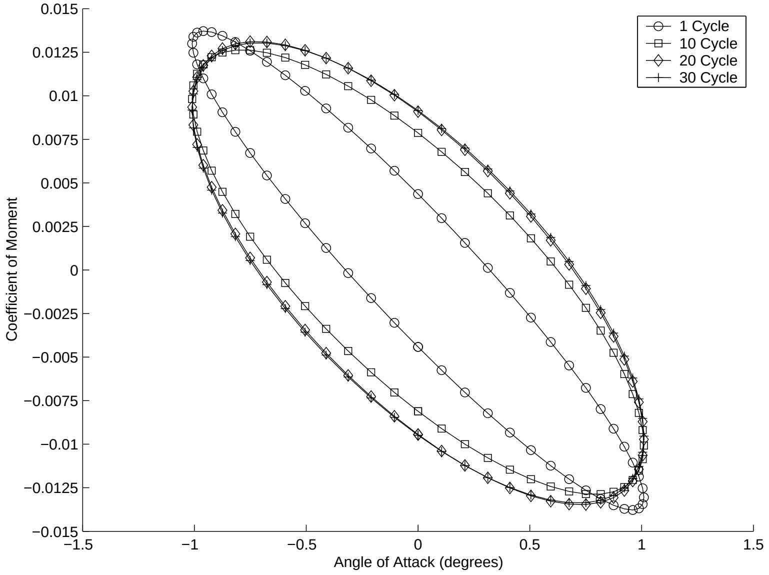


All C-mesh grid and mode permutations

Results - Pitching Airfoil - Euler Convergence

- Convergence results based on 129x33 C-mesh with 1 mode.





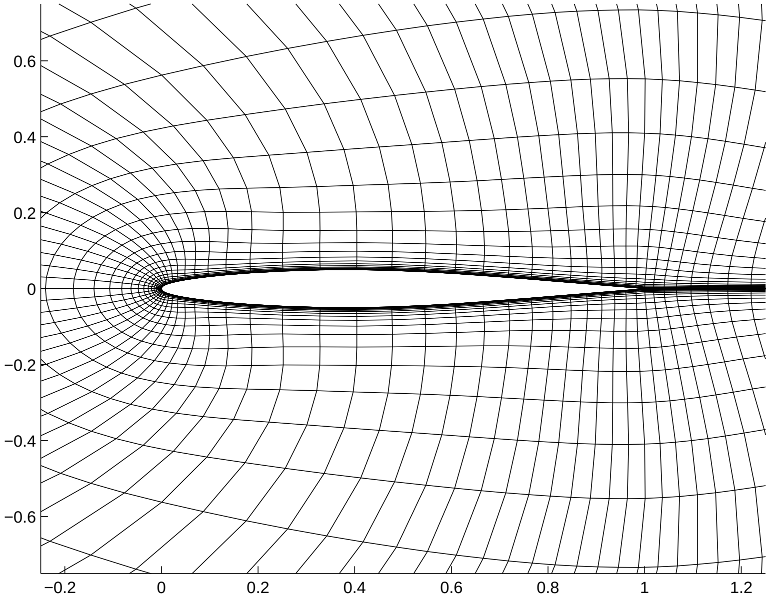
Results - Pitching Airfoil - Viscous

- The measured coordinates of the CT6 airfoil were significantly different than the theoretical 64A010 airfoil. Two sets of grids were generated based on the different geometries.

Topology	Dimensions	Boundary Distance (Chords)	Mean Grid Spacing at Wall (y+)
C-mesh	129x33	15	11.6
C-mesh	193x49	12	6.9
C-mesh	257x65	12	3.8

Table 3: Viscous Grid Configurations

Results - Pitching Airfoil - Viscous



129x33 C-mesh

257x65 C-mesh

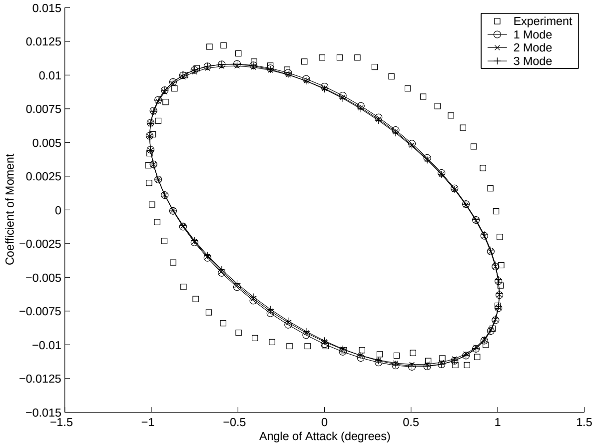
Results - Pitching Airfoil - Viscous - Cl

- Results are depicted for all grid/mode permutations.

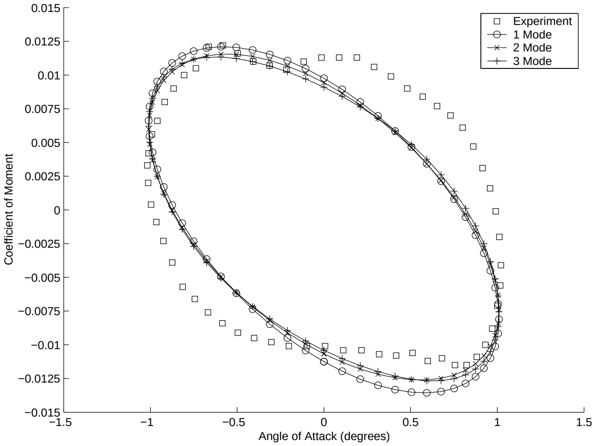
64A010 Coordinates

CT6 Coordinates

Results - Pitching Airfoil - Viscous - Cm

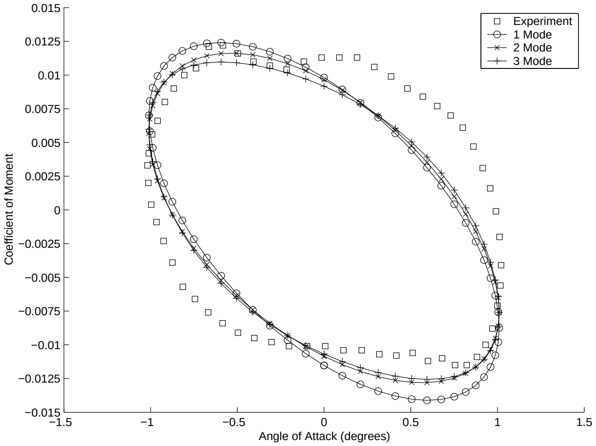


64A010 airfoil
129x33 grid at 3 separate modes



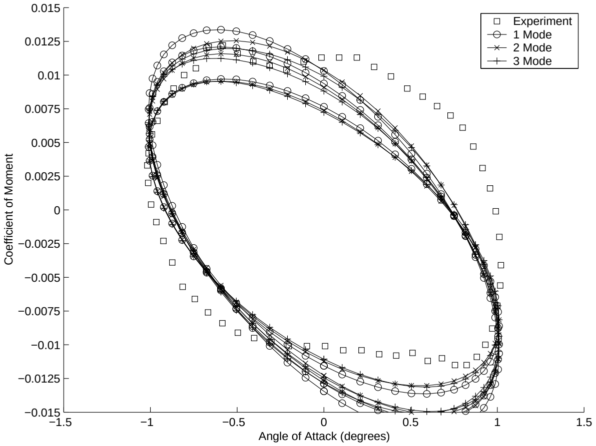
64A010 airfoil
193x49 grid at 3 separate modes

Results - Pitching Airfoil - Viscous - Cm



64A010 airfoil

257x65 grid at 3 separate modes

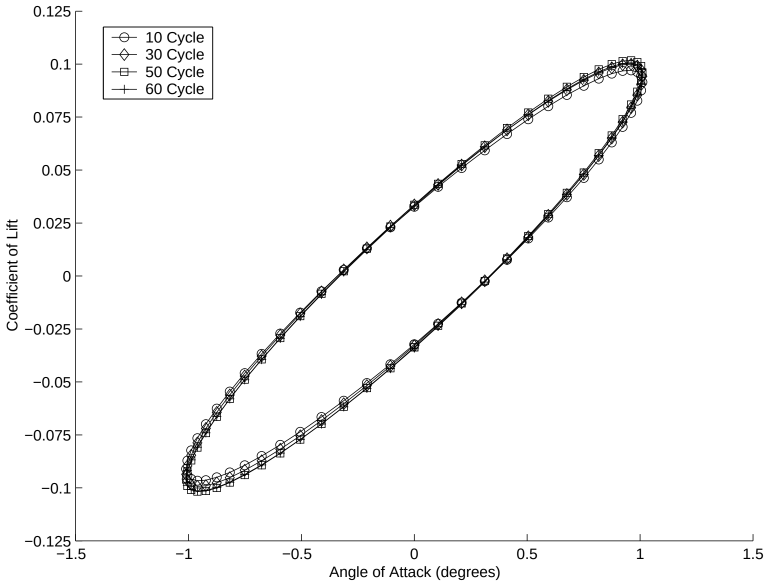


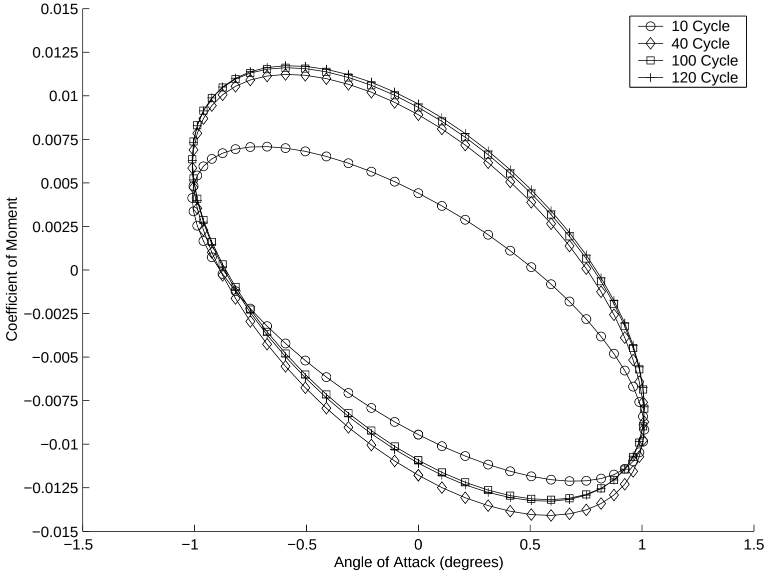
CT6 airfoil

All grid and mode permutations

Results - Pitching Airfoil - Viscous - Convergence

- Convergence results based on 193x49 C-mesh with 1 mode.





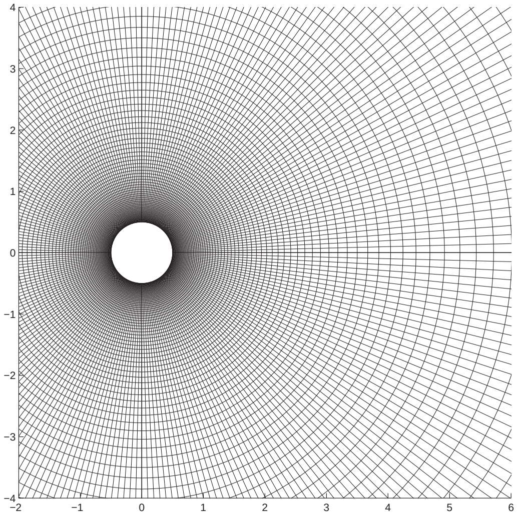
Results - Execution Time

- Timing results based on 129x33 Euler mesh and 193x49 viscous mesh.
- Results compiled on a 1.4Ghz AMD Athlon using 64 bit floating point math.

Mesh	1 Mode (secs/cycle)	2 Modes (secs/cycle)	3 Modes (secs/cycle)
129x33	0.59	1.0	1.5
193x49	1.9	2.9	4.5

Table 4: Execution Time - Euler and Viscous Meshes

Results - Cylinder



- The grid size is 256 by 128 cells. The mesh boundary is 200 chords from the center of the cylinder. An exponential stretching function is used in the radial direction with the smallest grid spacing of $3.54e-03$ chords occurring at the wall. At the top of the cylinder roughly 15 grid points captured the boundary layer.

Results - Cylinder - Fixed Time Period

Experiment	$-C_{pb}$	C_d	S_t
Williamson and Roshko (1990)	0.83		
Roshko (1954)			0.185
Wieselsberger (1922)		1.3	
Henderson (1995)	0.83	1.34	

Table 5: Time Averaged Experimental Data

Temporal Modes	$-C_{pb}$	C_d
1	0.832	1.257
3	0.895	1.306
5	0.903	1.311
7	0.903	1.311

Table 6: Previously Reported Time Averaged Data versus Temporal Modes

Results - Cylinder - Strouhal

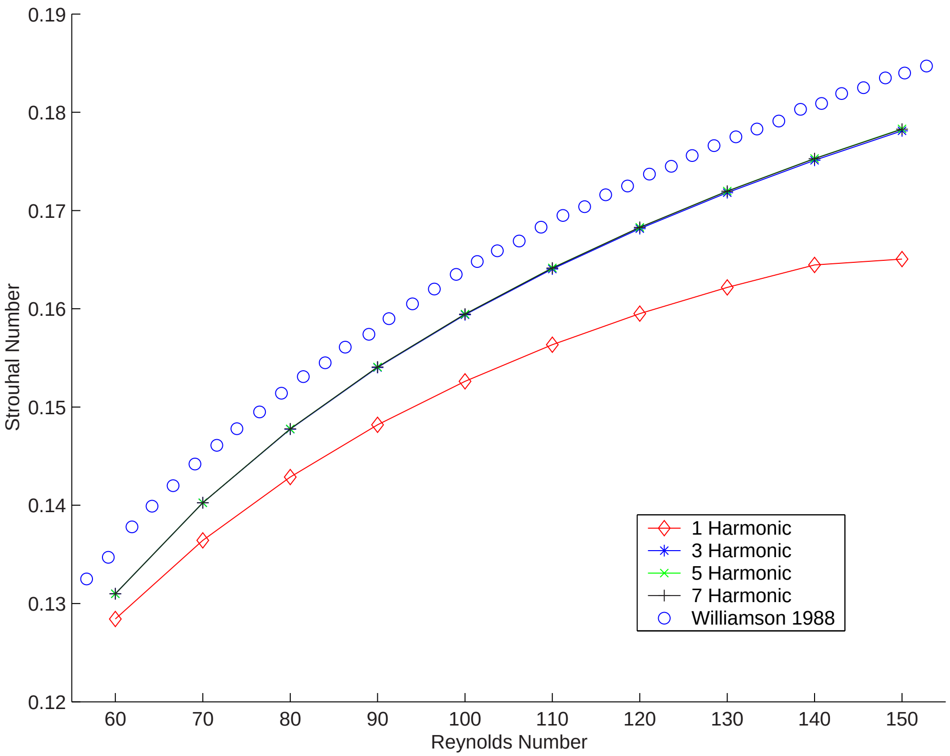


Figure 1: Strouhal Numbers versus Reynolds Number For Laminar Vortex Shedding

Results - Cylinder - Cpb

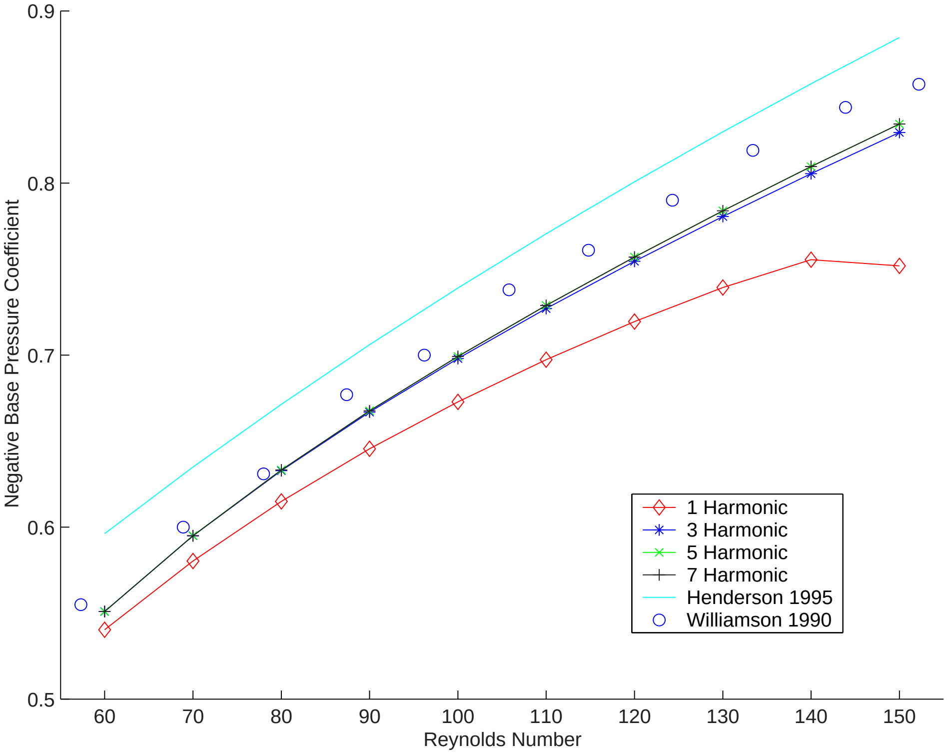


Figure 2: Mean Base Pressure Coefficient versus Reynolds Number For Laminar Vortex Shedding

Results - Cylinder - Cd

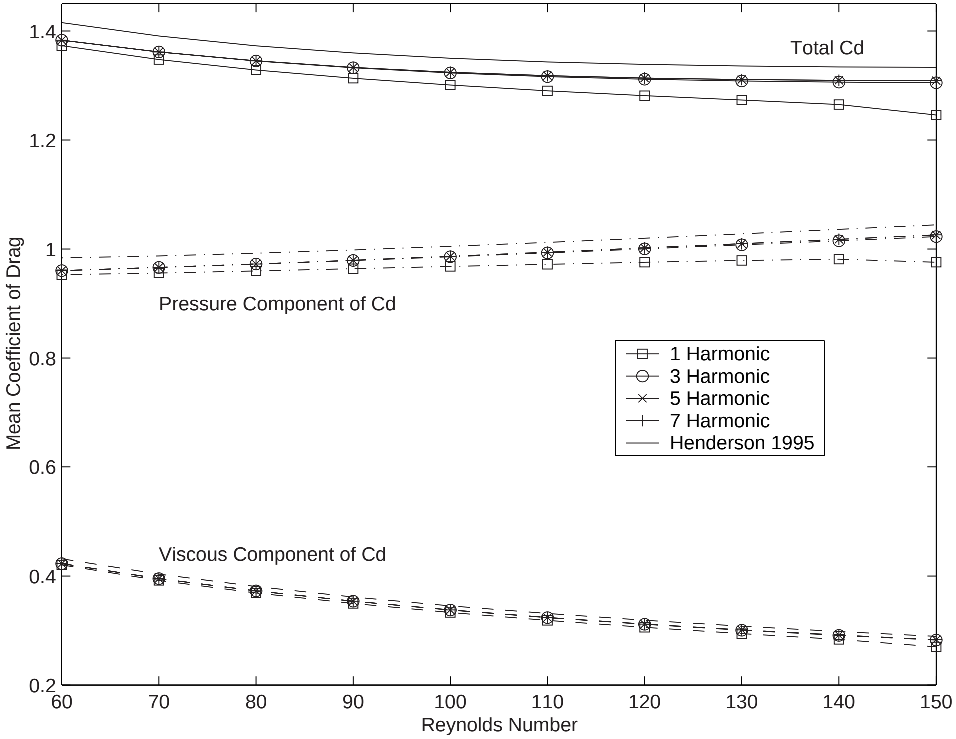


Figure 3: Mean Coefficient of Drag versus Reynolds Number For Laminar Vortex Shedding

Conclusions

- For the cylinder shedding test case, relatively accurate global coefficients were obtained using three temporal modes.

- For the forced pitching airfoil case only one temporal mode was needed to accurately predict the coefficient of lift. Experimental data for the coefficient of moment is poorly predicted.

- The dominant natural frequency can be predicted by the gradient based variable time period (GBVTP) method.

Acknowledgments

- Department of Energy - Accelerated Strategic Computing Initiative (ASCI)
- Kenneth C. Hall - Associate Professor in the Mechanical Engineering and Materials Science Department, Duke University
- Sanford Davis - NASA AMES Research Center
- Charles Williamson - Professor in the Mechanical and Aerospace Engineering Department, Cornell University

Future Work

- Continue verification process to turbomachinery cases where experimental data exists.
- Modify TFLO code to implement the nonlinear frequency domain method.