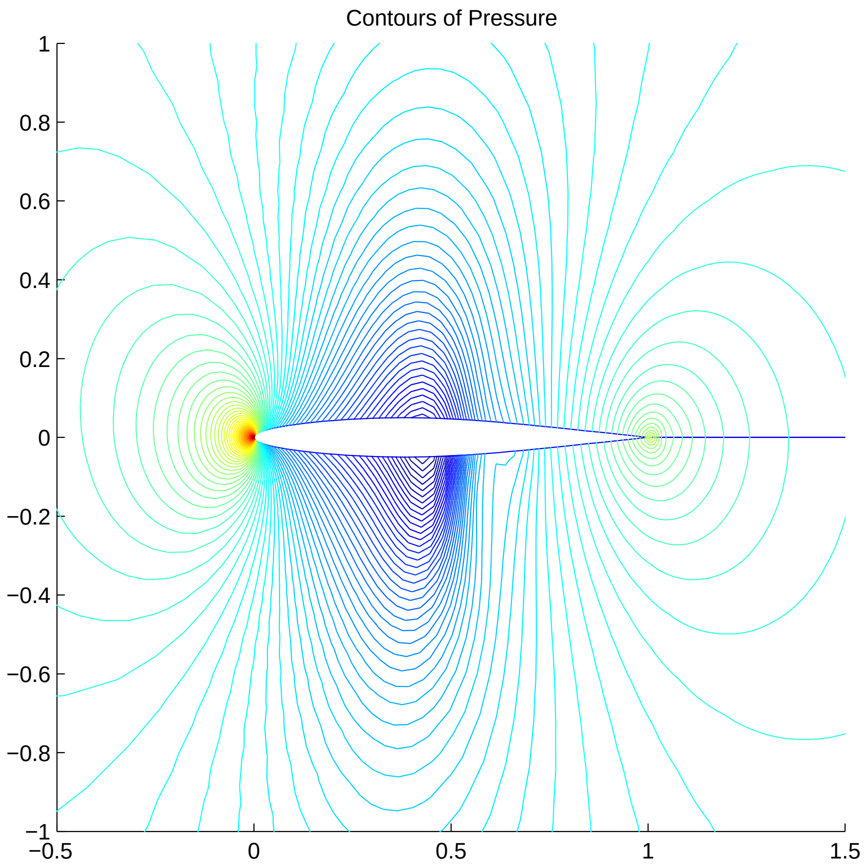
Application of a Non-Linear Frequency Domain Solver to the Euler and Navier-Stokes Equations



Matthew McMullen
Antony Jameson
Juan J. Alonso
Aero Astro Dept.
Stanford University

Stanford Affiliates
Palo Alto, CA
April 23-24, 2002

Motivation

- The goal of ASCI is to calculate the unsteady flow of an aircraft gas turbine engine. This includes component simulations of the compressor, combustor, and turbine.

Component	% Wheel	Total CPU Hours
Turbine	16	2.0 million
Compressor	16	5.4 million

- The job size typically varies between 500-1,000 processors, but estimates for job length are now provided in months.

- TFLO uses a dual time stepping algorithm based on a system of nested loops. The inner loop solves a time accurate system of equations typically requiring between 30-80 multigrid cycles. The outer loop advances the solution in time. A single oscillation in the solution uses 24-36 time steps requiring approximately 1000 multigrid cycles.

Governing Equations

- The Navier-Stokes equations can be written using simplified notation.

$$V\frac{\partial W}{\partial t} + R(W) = 0$$

- Expanding both W and $R(W)$ with a Fourier series in time

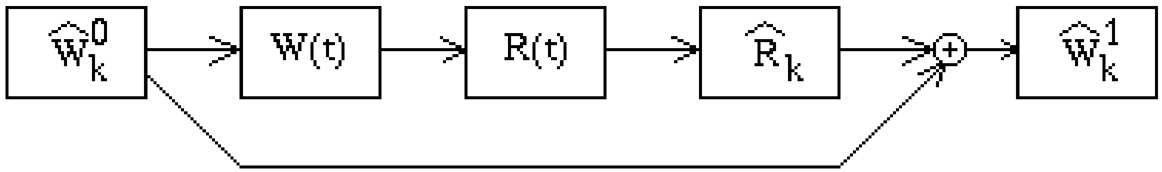
$$W = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{W}_k e^{ikt} \qquad R(W) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{R}_k e^{ikt}$$

- Add in a pseudo-time derivative and numerically integrate the equations.

$$V\frac{d\hat{W}_k}{d\tau} + ikV\hat{W}_k + \hat{R}_k = 0$$

Governing Equations

- Each iterative step in the solution process uses the following data flow.



- Since we are dealing with solving a steady system of equations, we apply established methods to accelerate the convergence.

  1. Multigrid V or W cycle
  2. Multi-stage RK scheme with local time stepping
  3. Residual averaging

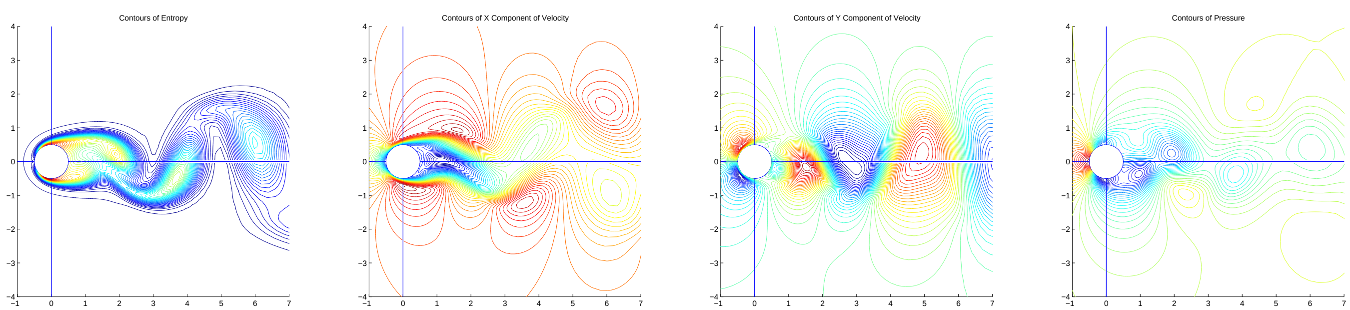
- We are dealing with real functions where the Fourier coefficients for the positive wavenumbers are equal to the complex conjugates of the Fourier coefficients for the negative wavenumbers. This eliminates computation required to integrate the negative wave numbers forward in pseudo-time.

Results

- Pitching Airfoil - Forced Frequency

    1. Euler
    2. Viscous
       Baldwin-Lomax Turbulence Model

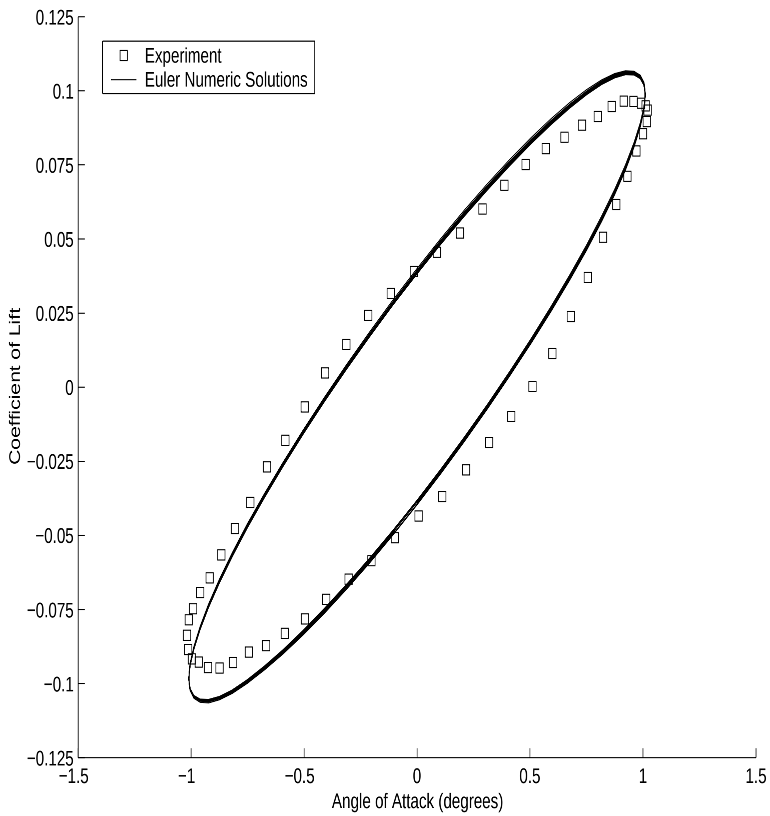
- Laminar vortex shedding from a cylinder. - Variable Frequency
  Utilizes a Gradient Based Variable Time Period method (GBVTP) to compute the Strouhal frequency.



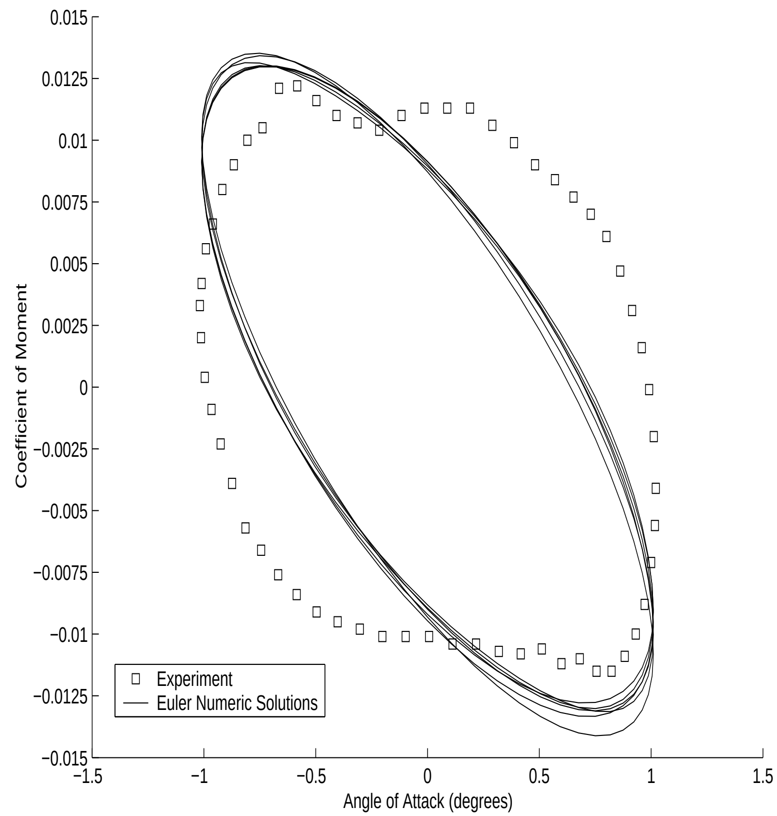
- Documented results for both cases are provided in AIAA-2001-0152 and AIAA-2002-0120.

Results - Pitching Airfoil
Euler - Coefficient of Lift and Moment

- Results used 1,2 and 3 temporal modes calculated on 4 different meshes ("O" and "C" mesh topologies)



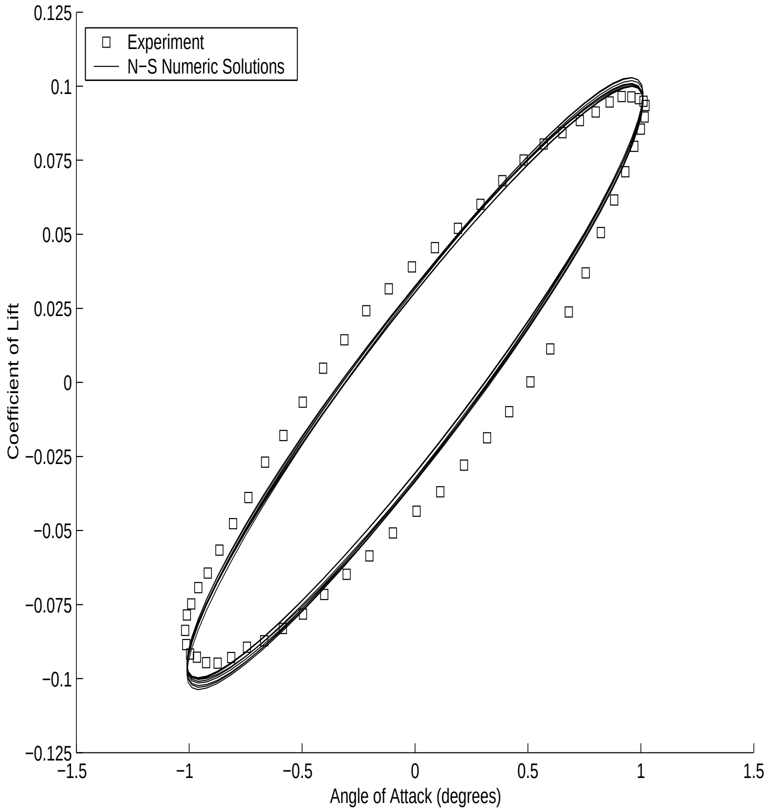
All grid and mode permutations



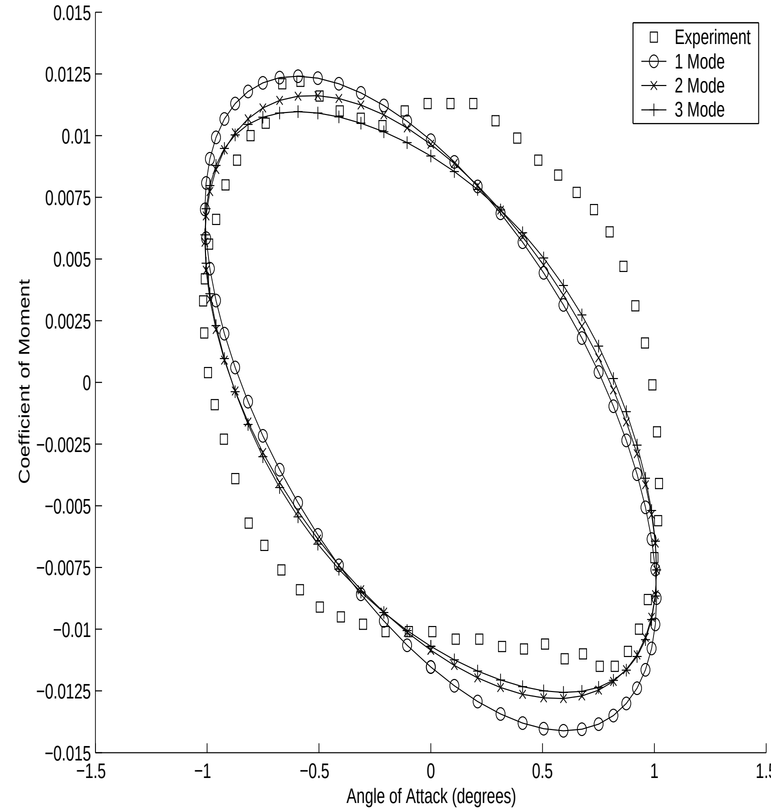
All C-mesh grid and mode permutations

Results - Pitching Airfoil
Viscous - Coefficient of Lift and Moment

- Results used 1,2 and 3 temporal modes calculated on 3 different meshes ("C" mesh topologies)



All grid/mode permutations

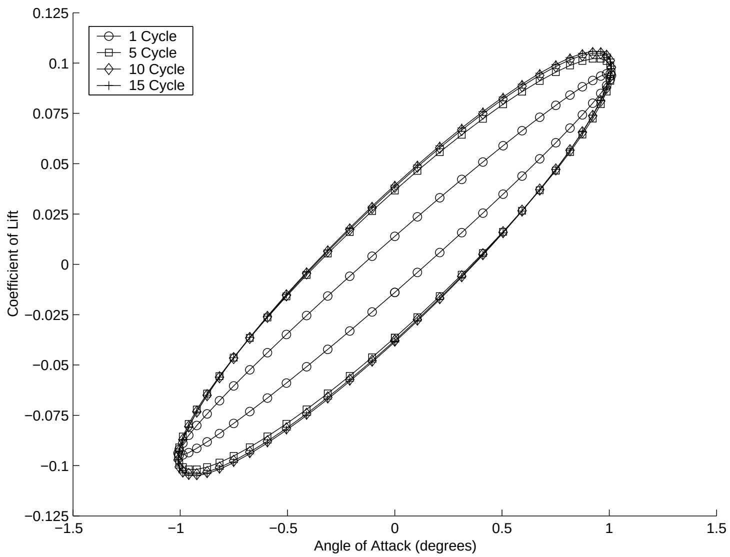


257x65 grid at 3 separate modes

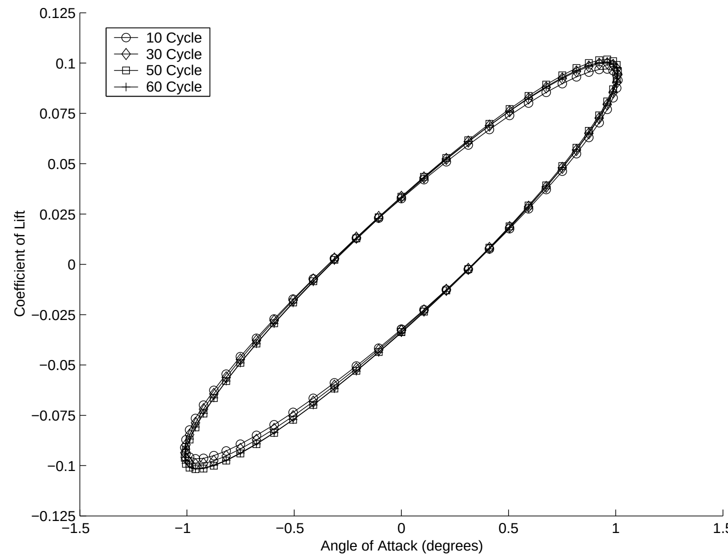
Results - Execution Time

- Timing results based on 129x33 Euler mesh and 193x49 viscous mesh. Results compiled on a 1.4Ghz AMD Athlon using 64 bit floating point math.

Mesh	1 Mode (secs/cycle)	2 Modes (secs/cycle)	3 Modes (secs/cycle)
Euler 129x33	0.59	1.0	1.5
Viscous 193x49	1.9	2.9	4.5



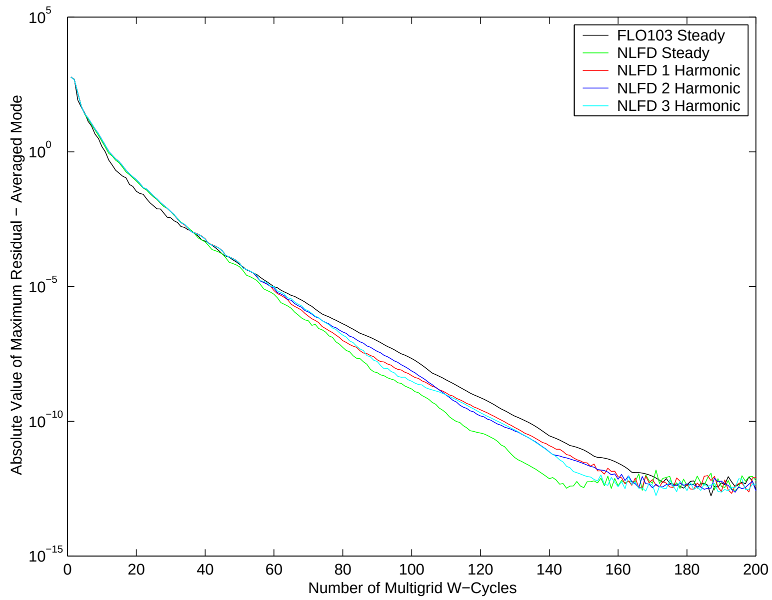
Euler 129x33



Viscous 193x49

Results - Pitching Airfoil
Convergence

- Convergence results based on a 193x49 Euler C-mesh.



	0th mode	1st mode	2nd mode	3rd mode
Normalized execution speed (time per cycle)	1.0	3.06	5.13	7.30

Conclusions

- For the cylinder shedding test case, relatively accurate global coefficients were obtained using three temporal modes. The dominant natural frequency can be predicted by the gradient based variable time period (GBVTP) method. Application of the GBVTP method has a positive influence on the solution of the conservation equations.

- For the forced pitching airfoil case only one temporal mode was needed to accurately predict the coefficient of lift. Experimental data for the coefficient of moment is poorly predicted.

- The computational cost of an unsteady solution is $2N + 1$ times the cost of a steady solution, where N is the number of unsteady modes.