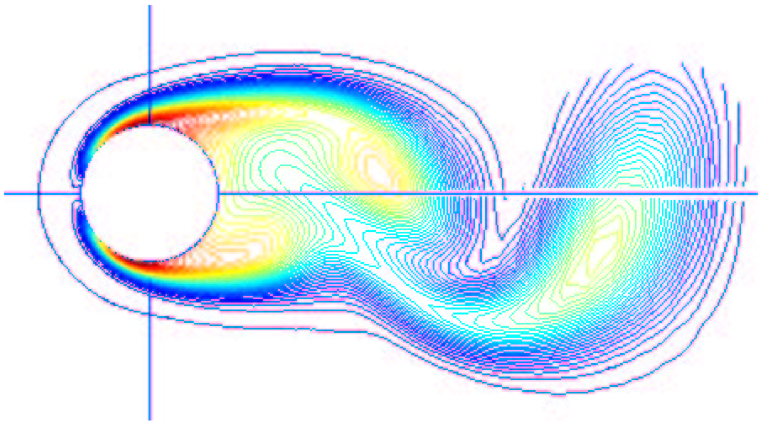
A Non-Linear Frequency Domain Method for Unsteady Flow Calculations



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Outline

- Motivation
- Potential benefits of the Non-Linear Frequency Domain method
- Development of the Non-Linear Frequency Domain equations
- Solution methods
- Verification

    1. Laminar vortex shedding for a cylinder
    2. Pitching airfoil

- Conclusion

Motivation

- Extreme CPU costs of unsteady flow calculations for turbo-machinery highlight the need for a more efficient method.

Component	Blade Rows	Grid Points (million)	% Wheel	Total CPU Hours (million)	Execution Duration
Turbine	9	94	16	2.0	250 days [1]
Compressor	23	N/A	16	5.4	675 days [1]

[1] Assuming 1000 processors running at 8 hours a day

- The dual time stepping scheme, in which inner iterations in pseudo time are used to solve a fully implicit backward difference formula (BDF) is still proving too costly in practice because of the need to use too many inner iterations.

- A representation in the frequency domain could be much less expensive.

Potential Benefits of the Non-Linear Frequency Domain Method

- In engineering applications it seems possible to obtain sufficient accuracy in practice with a small number of temporal modes, some-times only a single dominant mode, leading to very large savings in computational cost.

- The method could be used to obtain an approximate initial stationary periodic state of a multistage machine inexpensively, before starting a true time accurate simulation.

Governing Equations

- The Navier-Stokes equations can be written as

$$\frac{\partial w}{\partial t} + \frac{\partial f(w)}{\partial x} + \frac{\partial g(w)}{\partial y} = 0$$

$$w = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{bmatrix}$$

$$f = \begin{bmatrix} \rho u \\ \rho u^2 + p - \sigma_{xx} \\ \rho u v - \sigma_{xy} \\ \rho u H - u\sigma_{xx} - v\sigma_{xy} + q_x \end{bmatrix} \quad g = \begin{bmatrix} \rho v \\ \rho u v - \sigma_{xy} \\ \rho v^2 + p - \sigma_{yy} \\ \rho v H - u\sigma_{xy} - v\sigma_{yy} + q_y \end{bmatrix}$$

where σ_{xx}, σ_{xy} and σ_{yy} are the viscous stresses.

Non-Linear Frequency Domain Method

To illustrate the application of this method consider the Navier-Stokes equations

$$\frac{\partial w}{\partial t} + R(w) = 0 \qquad (1)$$

where R(w) is the discretized space residual

$$R(w) = D_x f(w) + D_y g(w)$$

(1) is transformed to

$$ik\hat{w} + \hat{R} = 0 \qquad (2)$$

where $\hat{w}$ is the transform of w(t), and $\hat{R}$ is the transform of R(t) = R(w(t))

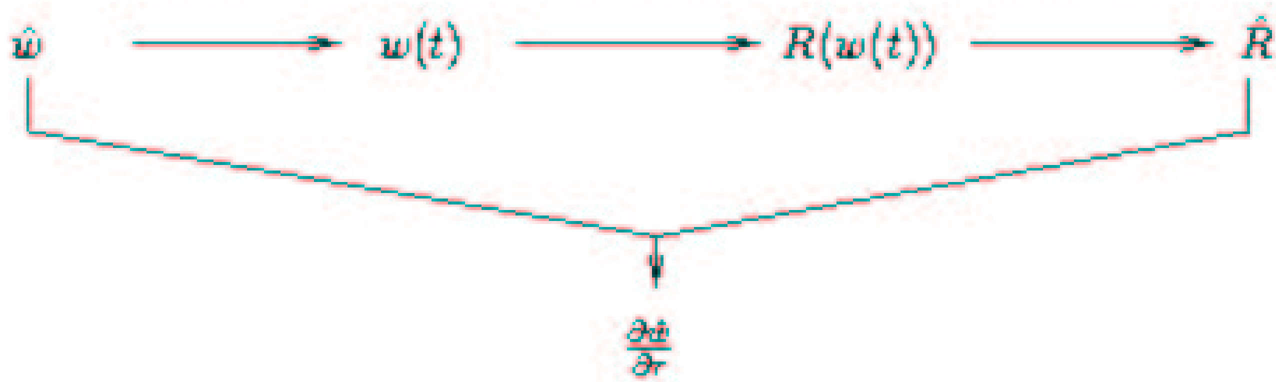
Non-Linear Frequency Domain Method

- Note that this requires the evaluation of R(t) by inserting the time history w(t) corresponding to $\hat{w}$ into the residuals.

- Given an initial guess w(t) solve (2) by an iterative procedure in pseudo time.

$$\frac{\partial \hat{w}}{\partial \tau} + ik\hat{w} + \hat{R} = 0 \qquad (3)$$

- A data flow diagram for each iterative step in the solution process is provided



Solution Techniques

- We are advancing a stationary system of equations in pseudo-time, we apply established methods to accelerate the convergence.

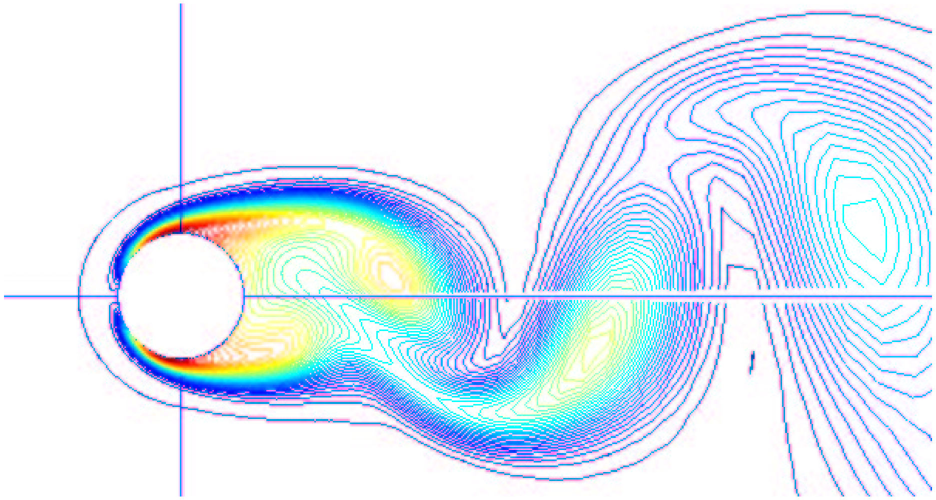
  1. Multi-stage RK scheme with local time stepping
  2. Implicit residual averaging
  3. Multigrid V or W cycle

- In addition, we are dealing with real valued functions where the Fourier coefficients for the positive wavenumbers are equal to the complex conjugates of the Fourier coefficients for the negative wavenumbers. Taking advantage of this property eliminates computing on half of the coefficients.

Laminar Vortex Shedding

- Two dimensional vortex shedding behind a cylinder occurs at Reynolds numbers between 49 and 194. The magnitude of the unsteadiness in the wake of the cylinder is significant in comparison to the mean flow values.

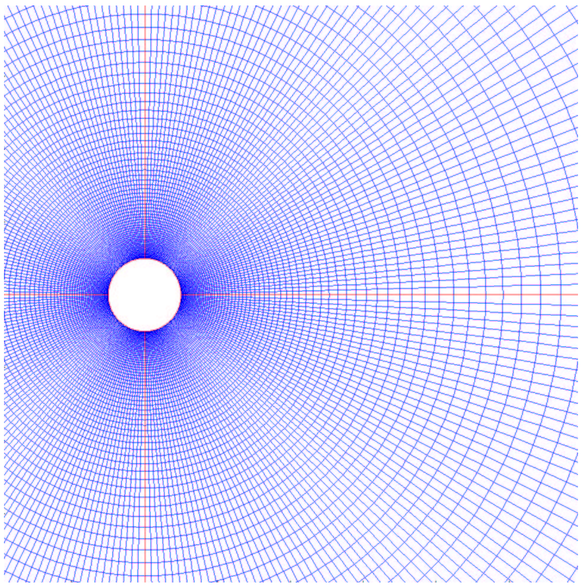
- A contour plot of entropy is provided to illustrate the rollup of vortices in the wake of the cylinder.



Laminar Vortex Shedding - Setup

- To illustrate the fully non-linear solution capabilities of this method we are solving this unsteady cylinder flow problem at a Reynolds number of 180.

- A near field plot of the grid and a table of its characteristics is provided below



Parameter	Value
Number of cells	256 x 128
Mesh boundary	200 chords from cylinder
Smallest spacing normal to wall	3.54 E-03 chords
Cells in boundary layer (12 o'clock position)	15

Laminar Vortex Shedding - Results

- Experimental results

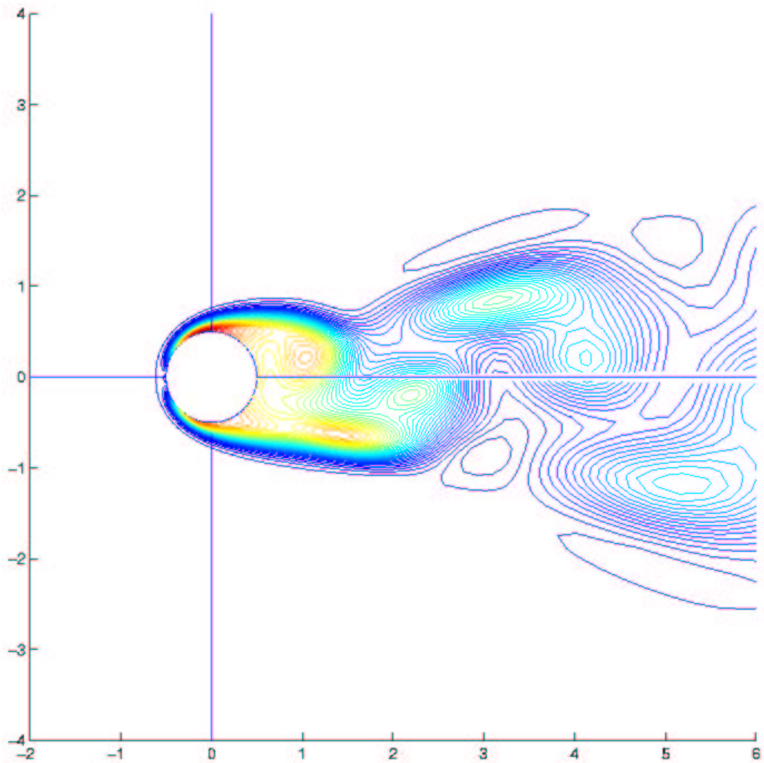
Experiment	$-C_{pb}$	C_d	S_t
Williamson and Roshko (1990)	0.83		
Roshko (1954)			0.185
Wieselsberger (1922)		1.3	
Henderson (1955)	0.83	1.34	

- Numerical results produced by frequency domain solver

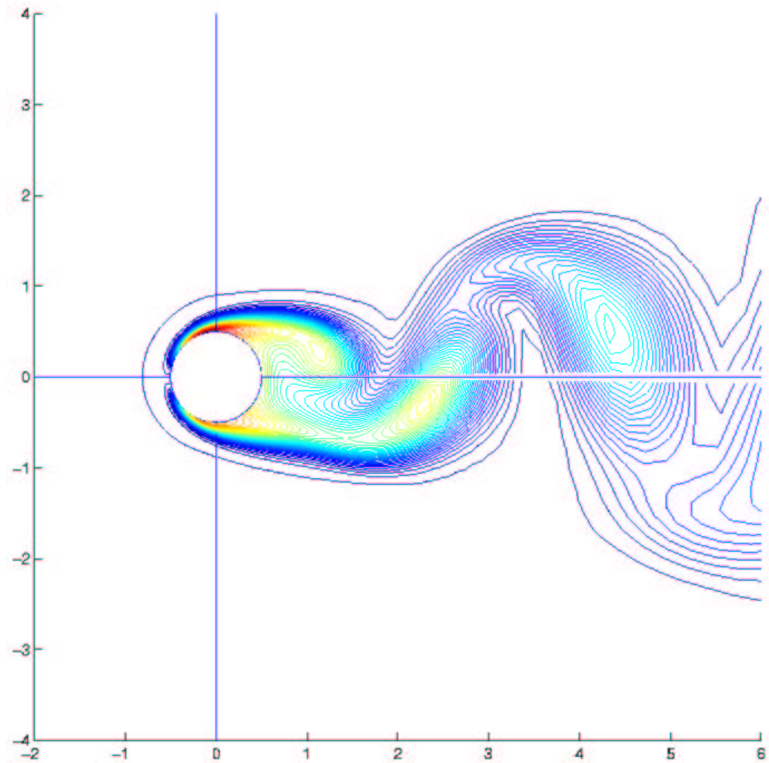
Number of Temporal Modes	$-C_{pb}$	C_d
1	0.832	1.257
3	0.895	1.306
5	0.903	1.311
7	0.903	1.311

Laminar Vortex Shedding - Results

- Contours of entropy in the cylinder wake



1 Temporal Mode

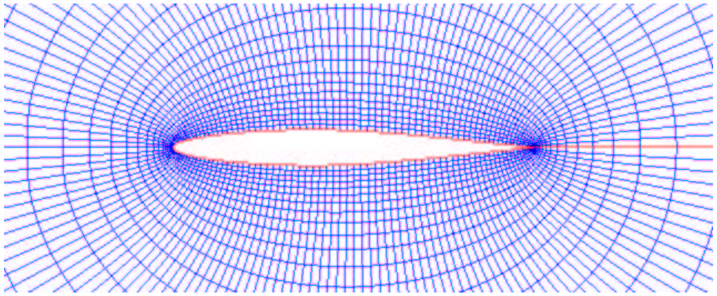


7 Temporal Modes

Pitching Airfoil

- Solutions of the Euler equations are presented. Unsteadiness is forced by a sinusoidal variation in angle of attack.

Parameter	Value
Angle of attack variation	1.01 degrees
Reduced frequency of oscillation[1]	0.212
Mach number	0.796
Grid size	161 x 33 points

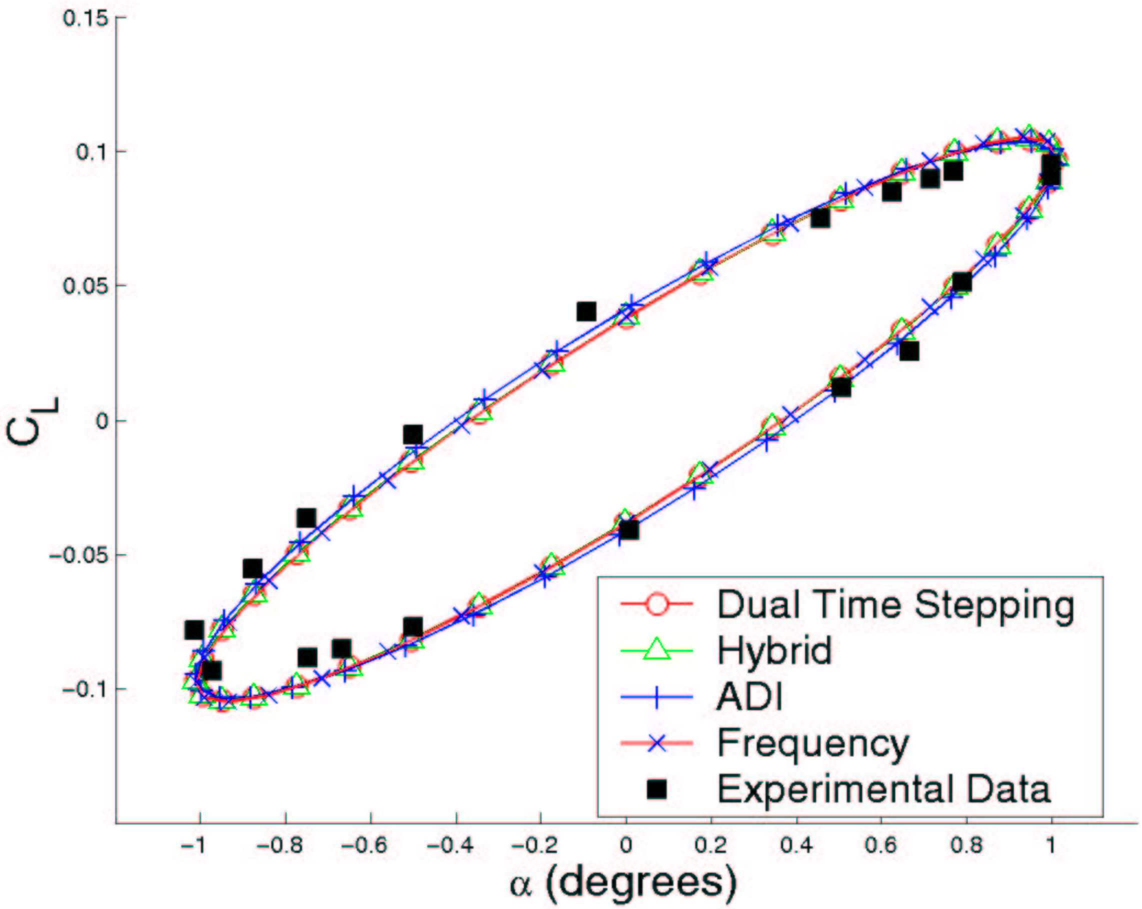


- Comparison to experimental results provided by S. Davis (1982) in AGARD Report 702 - CT Case No. 6, Dynamic Index 55, and to numerical simulations provided by J. Hsu.

[1] Reduced frequency is defined as $\frac{\omega Chord}{2q_{\infty}} = 0.212$

Pitching Airfoil - Results

- Comparison of the computed and experimental variation in the lift coefficient for the NACA 64A010 airfoil during a oscillation period.



Conclusions - Frequency Domain Method

- A Non-Linear Frequency Domain solver for the Navier-Stokes equations has been developed.

- The following test cases have been completed to verify the code
  1. Unsteady one-dimensional channel flow (not shown)
  2. Laminar vortex shedding behind a cylinder
  3. Pitching airfoil

- The results show good agreement with analytic and numerical solutions, and also experimental data, using a small number of modes.

- The method offers the possibility of drastically reducing the computational cost of unsteady simulations in turbo-machinery.