APPLICATION OF CONTROL THEORY IN AERODYNAMICS

A proposal for scientific research to AFOSR

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1. Background and Rationale

During the last two decades major advances have been realized both in control theory and in our capability to calculate complex aerodynamic flows. In control theory there have been far-reaching developments in the design of robust controls for systems with incomplete or noisy information, and uncertain or varying plant parameters [1, 2, 3] . Concurrently there has been the development of methods for treating distributed parameter systems, governed by partial differential equations. A rather complete theory for linear systems governed by partial differential equations is set forth in the book by Lions [4] . In aerodynamics the focus has been on the development of computational methods to calculate complex flows [5, 6]. Mathematical models of fluid flow are available at widely varying levels of complexity, ranging from Laplace's equation for ideal incompressible inviscid flow, the subject of classical theories of hydrodynamics, to the Navier-Stokes equations for incompressible and compressible viscous flows, and the Boltzman equations for rarified gas flows. The range of scales in a turbulent viscous flow is so large that resolution of the smallest scales remains beyond the reach of computational methods, except for some highly simplified configurations such as a box with periodic boundary conditions. Consequently viscous simulations of realistic configurations continue to rely on turbulence models to represent the effects of turbulence, with the prospect that it may become feasible in the coming decade to use subgrid scale models, which allow direct calculation of the large scale eddies. Transition to turbulence and flow separation off smooth surfaces are examples of aerodynamic problems which remain beyond the reach of currently existing methods. On the other hand the flight envelope of an aircraft largely encompasses regimes in which useful predictions can be made by simpler mathematical models. It is only at the extreme corners of the envelope, such as stall at high angle of attack, that the need to predict separation becomes predominant. In particular, inviscid transonic and supersonic flows over very complex configurations, ranging up to entire aircraft with flow through the engine ducts, can now be predicted quite accurately with manageable computer costs [7, 8].

In parallel with these advances in control theory and aerodynamics, rapid advances in computer technology have made it feasible to tackle increasingly complex problems. In the light of these developments, there now appears to be the potential to realize significant benefits by merging control theory and aerodynamics in several important problem areas. These can be distinguished broadly as:

1) vehicle control
2) flow control
3) aerodynamic design problems

These categories are not strictly exclusive and some problems can properly be regarded as belonging to more than one category.

Within the first class, the aerodynamic design of an aircraft and the design of its control system are strongly interdependent. Indeed, the success of the Wright brothers in developing the first viable aircraft may largely be attributed to their understanding of the control problems. More recently the introduction of sophisticated control technology has allowed the design of control configured vehicles (CCV) in which increased aerodynamic performance has been realized by sacrificing inherent aerodynamic stability. With currently existing theories the mathematical analysis of the control system is still carried out separately, with the airframe or plant characteristics assumed to be specified in advance. For example, given a linear system such as

$$\dot{x} = Ax + Bu$$

where x is the state vector, the control u would be optimized, normally as a feed back control, assuming that A and B are given. The ultimate realization of the CCV concept calls for a mathematical procedure for integrated airframe design and control optimization where $A = A(\mu)$, $B = B(\mu)$, and μ represents a set of parameters under the control of the designer which would be jointly optimized with u, but would be fixed during the design process and not varied during the operation of the vehicle.

Also within the first class of problems, there are important opportunities to reduce structure weight by the use of the control system for gust alleviation and flutter suppression. Gust alleviation systems have already been used to increase the fatigue life of large aircraft. Preconditions for acceptance of designs which depend on a flutter suppression system for their structural integrity will be total reliability and fail-safe design of the control system, and complete confidence in the underlying mathematical analysis. Aerodynamic forces in unsteady flow are subject to time delay effects due to the velocities induced by a trailing vortex system of fluctuating strength, corresponding to the fluctuating lift. The control system might be designed with mathematical models of varying complexity, but a least involving time dependent partial differential equations. Flutter would normally occur in the transonic regime in which shock waves are likely to have a strong influence. Therefore it would be desirable to use a nonlinear model of the flow. At least for airfoils and wings, solutions of the simpler nonlinear models, such as the transonic potential flow equation, can be computed fast enough and cheaply enough to contemplate their use in the design of a control system. The principal issue seems to be the computational feasibility of designing an effective control in a feedback form, suitable for closed loop control.

Within the second broad class of flow control problems there immediately comes to mind the possibilities of both active and passive boundary layer control to prevent or delay transition to turbulence. The possibility of delaying transition by distributed suction is well known. This is an example of active control, but without feedback. It can, however, be effective, and it is largely the manufacturing and operational problems such as production cost, surface accuracy, and maintenance of surface quality that have prevented its introduction and use. Issues arising in the development of a feedback control to prevent transition include the effectiveness of possible boundary controls (temperature, or compliant surface), what to measure and how, and the speed required for on-line computation. Passive control might be effected by using the pressure difference created by the flow to generate a bleed through a porous surface, say, on the underside of the wing. The shape may also be designed to delay transition: this belongs to the third category of design problems.

Another opportunity for flow control may be found in compressors for jet engines. Compressor performance is generally limited by blade stall leading to surge, beginning with the appearance of a rotating patch of stalled flow. It has been demonstrated that the onset of rotating stall can be delayed by active control of the blade pitch angle [9].

The third class, aerodynamic design problems, is the principal target of this proposal for research. Any aerodynamic device, such as an airplane wing or a ducted fan, may properly be regarded as a device for controlling the flow to realize some desired objective such as lift or thrust. Consequently the design of these devices may be considered as belonging to control theory, in particular the control of systems governed by partial differential equations with boundary control, where the form of the control is variation of the actual shape of the boundary rather than some input such as pressure. The shape might be varied during the operation of the system, as is the case with movable flaps or control surfaces on a wing, but such variations are generally feasible with only a few degrees of freedom, such as extension and angular movement of a flap. The basic shape, treated as infinitely variable, has to be fixed during the implementation of the design. This is essentially a form of off-line control. As such it can be calculated once and for all as an open loop control, and consequently the possibility is opened up of tackling quite complex non-linear problems which might otherwise be computationally infeasible.

This offers a route to bring CFD methods directly into the design process to find optimal aerodynamic shapes for a given set of objectives, whereas previously CFD methods have been largely limited to the analysis of flows over given shapes. If the analysis indicated that a given shape failed to meet the objectives, it was up to the designer to use his intuition and experience to devise another shape. Such a process of trial and error is typically both lengthy and difficult, with no assurance of the optimality of the final design. Formulation of the design problem in the framework of control theory reduces the design process to a systematic procedure which can be implemented on a computer. Preliminary studies by the author, reported in [10, 11, 12], have demonstrated the promise of this approach. In particular, it was demonstrated that two dimensional wing sections can be automatically redesigned to improve their transonic performance by reducing or eliminating shock waves which would otherwise appear in the flow. It was also demonstrated that airfoils could be optimized for their best average performance over several design points at different flight conditions. In another (unpublished) example, the method was used to design hydrofoil sections which could achieve higher speeds before the onset of cavitation by limiting the peak suction over a range of lift coefficients.

In these examples the computational complexity was reduced by the restriction to both two dimensional and potential flow. The computational costs were further contained by the use of sophisticated numerical algorithms, both for the calculation of the flow and for the solution of the adjoint equation needed for implementation of the. control. A typical redesign of a transonic airfoil with, say, three design points, can be carried out in about 20 minutes on a Convex C2 mini-supercomputer. Therefore it appears feasible to extend the method to a variety of more complex design problems involving either more complex and complete mathematical models of the flow, or more complex geometric configurations, in both two and three dimensional applications.

The main thrust of this proposal is to pursue the application of control theory for aerodynamic design in a broad investigation which will cover both the study of a variety of mathematical, algorithmic and technical issues which underlie the formulation of the method, and its systematic extension to progressively more complex applications. These are outlined in the following sections. When one considers the range of problems in which shape design is important including, for example, wave resistance of ships, or electromagnetic signatures of aircraft, it can be envisioned that this could be the nucleus of a very broad program of research with increasingly far-reaching applications in the future. At the same time, while proceeding on this path, problems in the first two classes ought not to be overlooked, and it is proposed that they be the subject of ongoing study to identify computationally feasible examples.

2. Formulation and Outline of the Theory

Objectives for computational aerodynamics can be broadly identified and distinguished at three levels:

1. Capability to predict the flow past the configuration, say an aircraft or a propulsive system, at all points of the operating envelope such as cruise, take off or maneuver.

2. Interactive calculations, possibly for components, that can be performed fast enough to allow rapid improvement of the design.

3. Automatic design optimization.

Control theory offers a route to the realization of the third objective, assuming that the designer is able to quantify the performance objectives with a cost function. For a wing section or a wing, for example, the cost function might be defined as

$$I = \frac{1}{2}\beta_1 \int (p - p_D)^2 \, dS + \beta_2 C_D + \frac{1}{2}\beta_3 \left(C_L - C_{L_D}\right)^2$$

where the integral is over the surface, p is the pressure and p_D is the desired pressure , C_D is the drag coefficient, C_L is the lift coefficient and C_{L_D} is the desired lift coefficient, while $\beta_1, \beta_2, \beta_3$ are parameters determining the relative importance of the three terms. The choice of a cost function requires careful thought, and it must be related to the mathematical model of the flow which is used for the calculation. In inviscid flow a non-lifting flat plate has no drag, so the drag coefficient by itself is not an adequate measure. In fact, without any specification of requirements on the desired pressure distribution, there is a risk of producing designs with strong adverse pressure gradients, which would result in separation of the boundary layer.

One approach to design is to try to solve the inverse problem of determining the shape which corresponds to a specified pressure distribution. [13, 14] Since, in general, no such shape necessarily exists, a direct formulation of the inverse problem can easily be ill posed. Control theory avoids this difficulty while also providing much more flexibility in the specification of the design goals, including the possibility of multiple design points.

The underlying concepts of the formulation can be clarified by considering first the case of optimizing a system defined by a set of algebraic equations. Suppose that one wishes to minimize $I(\phi,f)$ where ϕ_j, $j=1,n$ represents the solution vector, f_k, $k=1,m$ represents the control vector, and the system is governed by nonlinear equations of the form

$$F_i(\phi,f) = 0, \qquad i = 1, n \qquad \text{(F)}$$

using the convention that a repeated index implies a summation over the range of that index, then to first order that the result of a modification δf in the control is

$$\partial I = \frac{\partial I}{\partial \phi_j}\delta\phi_j + \frac{\partial I}{\partial f_k}\delta f_k \qquad \text{(PC)}$$

where the $\delta\phi_j$ can be determined from the perturbation equations

$$\frac{\partial F_i}{\partial \phi_j}\delta\phi_j + \frac{\partial F_i}{\partial f_k}\delta f = 0 \qquad \text{(PF)}$$

Now (PF) might be solved to give the influence coefficients $\frac{\partial \phi_j}{\partial f_k}$, but this is an unnecessarily large volume of information. To eliminate the need to determine $\delta\phi_j$ explicitly one can multiply (PF) by Lagrange multipliers or costate variables ψ_i, and subtract from (PC) to obtain

$$\delta I = \left(\frac{\partial I}{\partial \phi_j} - \psi_i \frac{\partial F_i}{\partial \phi_j}\right)\delta\phi_j + \left(\frac{\partial I}{\partial f_k} - \psi_i \frac{\partial F_i}{\partial f_k}\right)\delta f_k$$

Now let the costate vector ψ satisfy the adjoint equation

$$\frac{\partial F_i}{\partial \phi_j}\psi_i = \frac{\partial I}{\partial \phi_j}, \quad j = 1, n \qquad \text{(A)}$$

Then the first term is eliminated and

$$\delta I = g_k \delta f_k$$

where the gradient vector g is defined as

$$g_k = \frac{\partial I}{\partial f_k} - \psi_i \frac{\partial F_i}{\partial f_k}$$

Now if one chooses a modification in the control

$$\partial f_k = - \lambda_k g_k$$

then to first order the change in the cost function is

$$\delta I = - \lambda_k g_k{}^2$$

guaranteeing an improvement if each λ_k is sufficiently small and positive.

The matrix of the adjoint equation (A) is the transpose of the matrix $\frac{\partial F}{\partial \phi}$ appearing in the perturbation equation (PF), and the cost of solving the adjoint equation, which is linear, will be no greater than the cost of solving the nonlinear system equation (F). Thus the gradient for an arbitrary number of control parameters can be obtained at a cost no greater than the cost of two solutions of the system equations. One may also regard this process as an indirect method of substituting

$$\delta \phi = \left[\frac{\partial F}{\partial \phi}\right]^{-1} \frac{\partial F}{\partial f} \partial f$$

in (PC) and solving in advance the equation

$$\psi^T = \frac{\partial I}{\partial \phi}^T \left[\frac{\partial F}{\partial \phi}\right]^{-1}$$

The corresponding formulation for systems governed by partial differential equations can be obtained by translating the same sequence of steps into operations with differential operators. The principal complication is the need to distinguish equations holding in the interior of the domain from boundary conditions. It is convenient to introduce separate inner products

$$(u, v)_D = \int_D uv dV$$

and

$$(u,v)_B = \int_B uv dS$$

defined over the domain D and the boundary B. Suppose that $I(\phi,f)$ is defined as an integral over the boundary such that the result of a modification δf in the control can be represented as

$$\delta I = \left(\frac{\partial I}{\delta \phi}, \delta\phi \right)_B + \left(\frac{\partial I}{\partial f}, \delta f \right)_B$$

and that the governing equation is

$$L(\phi,f) = 0 \qquad \text{(F*)}$$

where L is a differential operator, with an appropriate boundary condition. The perturbation equation is now

$$A\delta\phi + C\delta f = 0 \qquad \text{(PF*)}$$

where $A = \frac{\partial L}{\partial \phi}$, $C = \frac{\partial L}{\partial f}$. Then introducing a costate vector ψ,

$$\delta I = \left(\frac{\partial I}{\partial \psi}, \delta\phi \right)_B + \left(\frac{\partial I}{\partial f}, \delta f \right)_B - (\psi, A\delta\phi)_D - (\psi, C\delta f)_D$$

Now let A* be an adjoint differential operator satisfying a relation of the form

$$(u, Av)_D - (A^*u,v)_D = (u, Bv)_B - (B^*u,v)_B$$

where B and B* are boundary operators. For example, in the case of ideal fluid flow governed by Laplace's equation

$$\int_D u\nabla^2 v\,dV - \int_D v\nabla^2 u\,dV = \int_B u\frac{\partial v}{\partial n}ds - \int_B v\frac{\partial u}{\partial n}ds$$

and $A=A^* \equiv \nabla^2$, while $B = B^* \equiv \frac{\partial}{\partial n}$. Then if the costate vector ψ is required to satisfy the adjoint equation

$$A^*\psi = 0 \quad \text{in } D \qquad (A^*)$$

the formula for δI only contains $\delta\phi$ in boundary integrals, which can be eliminated by the choice of appropriate boundary conditions, yielding a direct relationship between δI and δf. From this the gradient g can be determined, such that the change in the cost can be expressed as

$$\delta I = (g, \delta f)_B$$

Again an improvement is assured by taking

$$\delta f = -\lambda g$$

with λ small enough and positive.

The complete set of equations and boundary conditions needed for implementation of this procedure are set forth in detail in reference [10] for three cases:

1. Two dimensional transonic potential flow over an airfoil generated by conformal mapping from a circle, using the boundary value of the modulus of the mapping function as the control

2. Airfoil design in two dimensional inviscid flow modelled by the Euler equations

3. Wing design in three dimensional inviscid flow modelled by the Euler equations

Computational experiments with the application of the method to the first of these three cases are reported in reference [11]. The results reinforce the belief that control theory can provide the basis of an effective tool for aerodynamic design. An example of the redesign of an airfoil to reduce its transonic shock induced drag is shown in Figure 1. This airfoil was originally designed to provide good subsonic performance at a lift coefficient in the range of .6, with relatively mild adverse pressure gradients to avoid separation of the boundary layer. Control theory was used to optimize the modified airfoil for two criteria. One was to minimize the drag at Mach .72, while the other was to minimize the deviation from the original pressure distribution at Mach .2 in order to preserve the mild adverse pressure gradients. After 8 design cycles the transonic drag was reduced from .0191 to .0001, at the cost of producing a slightly peaky subsonic pressure distribution near the leading edge, but with no impairment of the mild pressure gradient on the rear upper surface.

3. Open Technical Issues for Research

While the feasibility of using control theory for aerodynamic design has been established in the preliminary studies that have already been carried out [10, 11, 12], further research is needed to develop both the underlying theory and the computational techniques to the fullest extent. Some of the open mathematical and algorithmic issues which could have a significant impact are outlined in the following paragraphs.

(A) The effectiveness of the method critically depends on the ability to determine the gradient (Frechet derivative) of the cost function with respect to the boundary shape. The method will be impaired if either

(1) The accuracy of the gradient is too badly contaminated by discretization and solution errors

or

(2) the computational cost of calculating the gradient is too large

One important issue is which of the following two procedures is to be preferred:

(1) to derive the adjoint equation in differential form with appropriate boundary conditions (equation A*), and then derive separate discrete numerical approximations to the flow and adjoint equations.

or

(2) to obtain first a discrete approximation to the flow equations, and then treat this approximation as an algebraic system with a corresponding algebraic adjoint equation, following the procedure described in the previous Section by equations (F), (PF) and (A).

In the initial studies [10, 11, 12] the differential approach (1) has been used. The numerical schemes must still respect the properties of both the flow and the adjoint equation. For example, if an upwind scheme is used for the flow equations, a downwind scheme is appropriate for the adjoint equation because of the reversal of the characteristic directions. The algebraic approach (2) will produce equations which may be regarded as a particular discretization of the adjoint differential equation and boundary conditions. It may yield a more precise gradient of the discrete system at the cost of a substantial increase in computational complexity. It may also tend to obscure the importance of correct boundary conditions to produce a well posed problem.

(B) Another issue to be explored is the possibility of simplifying the gradient calculation by basing it on a simplified mathematical model of the flow. For example, while using the Euler equations to model an inviscid flow, one might base the gradient calculation on a potential flow model with a correspondingly simplified adjoint equation.

(C) Given a satisfactory procedure for calculating the gradient, the efficiency of the method might be greatly improved by the use of accelerated descent procedures. These could include

(1) the use of line searches to find the minimum in each search direction

(2) the use of a generalized conjugate gradient procedure for nonsymmetric systems.

(D) The way in which the controlling boundary shape is parameterized may also have a critical influence on the efficiency of th method. The use of conformal mapping to define the shape has proved to be extremely effective in two dimensional airfoil design [11], particularly in giving good control of the critical regions in the neighborhood of the leading and trailing edges. On the other hand, the use of a simple surface displacement as the control provides no built-in mechanism to assure a high degree of smoothness in the designed shape, and is likely to be much less effective. Systematic studies will be needed to find effective parameterizations for three dimensional problems.

(E) In order to produce useful or even physically relevant designs, various geometric and structural constraints must be satisfied such as

(1) closure of the trailing edge and wing tip.

(2) prevention of cross-overs in the profile in both the chordwise and spanwise directions.

(3) minimum thickness or volume requirements for structural strength and fuel storage.

The design procedure must be formulated to allow for these constraints. For example, in the case of airfoil design for minimum drag with conformal mapping as the control [11], the gradient is dominated by terms which tend to produce a crossed-over trailing edge, since such a shape has a larger projected area in the forward direction than in the backward direction, producing negative drag. Essential constraints of this kind can be satisfied by projecting the successive shape modifications into an allowable subspace. Softer constraints, such as a requirement on contained volume, may more easily be addressed by the incorporation of appropriate penalty functions in the cost function.

(F) Regularization procedures are needed to allow both for geometric singularities such as the corner at a sharp trailing edge, and singularities in the flow such as shock waves and vortex sheets. Generally these take the form of smoothing procedures, which may be introduced both through the choice of more sophisticated cost functions based on Sobolev norms, and directly in the descent procedure.

For example, in designing an airfoil for a specified pressure distribution one might take the cost function to be

$$I = \frac{1}{2} \int_B (p - p_D)^2 ds$$

where p is the actual surface pressure, p_D is the desired surface pressure, and s is the arc length along the profile. In transonic flow both p and p_D might be discontinuous due to the presence of shock waves, leading to discontinuous shape modifications. To avoid this difficulty one can define the cost function to be

$$I = \frac{1}{2} \int_B \left(S^2 + \varepsilon \left(\frac{dS}{ds} \right)^2 \right) ds$$

where S satisfies

$$\frac{d}{ds} \varepsilon \frac{dS}{ds} + S = p - p_D$$

This eliminates a discontinuity in the gradient due to a discontinuity in $p - p_D$.

To introduce smoothing directly into the descent procedure one can define the shape change δf by the equation

$$\varepsilon_1 \delta f - \frac{\partial}{\partial s} \varepsilon_2 \frac{\partial}{\partial s} \delta f = -g$$

To first order the change in the cost is

$$\delta I = \int_B g \delta f ds$$

Substituting for g and integrating by parts

$$\delta I = - \int_B [\varepsilon_1 \delta f^2 - \delta f \frac{\partial}{\partial s} (\varepsilon_2 \frac{\partial}{\partial s} \delta f)] \, ds$$

$$= - \int_B [\varepsilon_1 \delta f^2 + \varepsilon_2 (\frac{\partial}{\partial s} \delta f)^2] \, ds$$

so an improvement is assured if e_1 and e_2 are both positive and sufficiently large.

(G) Optimization with a poorly chosen cost function will generally lead to a correspondingly poor result. The cost function must be carefully designed both

(1) to assure a well-posed problem, avoiding pitfalls such as using a lengthy numerical calculation to find that a nonlifting flat plate in inviscid flow has zero drag

and

(2) to realize design benefits with proper trade-offs.

Systematic studies are needed to determine the effect of including different terms in the cost function, and their relative influence as their weight is varied.

(H) In order to achieve really useful design trade-offs it will also be essential to develop a multi-point design capability. For example, one may wish to reduce transonic drag in a cruising condition at moderate lift, while also trying to improve the high lift properties of the configuration for take-off and landing. In principle one can define the cost function as a sum of measures at Q different design points

$$I = \sum_{k}^{Q} I_k$$

and obtain the gradient as a corresponding sum

$$g = \sum_{k=1}^{Q} g_k$$

This procedure has proved successful for airfoil design [11]. The influence of multiple design points on the convergence of the descent procedure needs to be investigated systematically.

It is proposed in this research project to investigate the issues outlined in the preceding paragraphs (A-H) in the context of a systematic program to extend the range of applications of the method. As more experience is gained with some of these applications, which are outlined in the next Section, it will become clearer which are the most important technical issues, which ought to be emphasized in the research.

4. Development Paths for Applications of Control Theory to Design

Several general directions may be distinguished as paths for extending the range of applications of control theory to aerodynamic design. Broadly these are

(A) Incorporation of more complete mathematical models of the flow.

(B) Treatment of different flow regimes or types of flow.

(C) Extensions to more complex geometric configurations in both two and three dimensions.

Some possibilities in each of these directions are outlined in the following paragraphs.

There is a hierarchy of mathematical models of fluid flow which can be considered for aerodynamic design. For compressible flow the most appropriate models in ascending order of complexity are

(1) Transonic potential flow

(2) Euler equations for inviscid flow

(3) Transonic potential flow with viscous boundary layer correction

(4) Euler equations with viscous boundary layer correction

(5) Navier-Stokes equations with Reynolds averaging of small scales and a turbulence model.

Since the design procedure will require some number of iterations which cannot readily be predicted, the computational complexity of the Navier-Stokes equations discourages their use in the early stages of the investigation. It appears more practical to concentrate on the potential flow and Euler models, and to try to build on the experience which has already been obtained. The introduction of boundary layer corrections is feasible in principle within the framework of control theory, and could enhance the practical value of the design method.

Computational methods have reached a level of maturity that might justify their use in design for subsonic, transonic and fully supersonic flows. Applications which appear to offer the possibility of a large pay-off can be found in both transonic flow, where there is a need to delay the onset of drag rise, and supersonic flow, where the feasibility of supersonic cruise depends on attaining a high enough lift-to-drag ratio. Modelled by either the potential flow equation or the Euler equations, the computational cost of calculating a steady supersonic inviscid flow past a slender sharp-nosed configuration is much lower than the cost of a transonic flow calculation because the equations are fully hyperbolic, and can be solved by streamwise marching in space [15, 16]. The corresponding adjoint equation for implementation of the control theory could also be solved inexpensively by upstream marching. In fact, if the equations are simplified to the case of linearized supersonic flow, the adjoint equation corresponds closely to the reverse flow equation, and the results of R. T. Jones for shapes of minimum drag [17] can be recovered by applying control theory. Because of the availability of solution methods using space marching, there appears to be an attractive opportunity to use control theory to optimize supersonic wings, possibly subject to constraints on the contained volume.

Intermediate between transonic and supersonic flow there is the special case of conical flow, which is an appropriate model for highly swept and delta wings, based on the assumption that the flow is invariant along rays from the apex. This reduces the calculation of a three dimensional supersonic flow to the calculation of a two dimensional transonic flow in a spherical cutting surface, and has been found to provide an effective and inexpensive tool for aerodynamic analysis [18]. Conical flow can usefully be modelled at the level of either potential flow or the Euler equations.

The following sequence is a representative hierarchy of aerodynamic problems of increasing geometric complexity.

(1) Two dimensional airfoil design

(2) Multi-element design of airfoils with slats and flaps

(3) Three dimensional wing design

(4) Wing-body design

(5) Wing-pylon-nacelle design

(6) Complete aircraft

There is a corresponding hierarchy for internal flows in jet engines, including two and three-dimensional cascades, rotors, rotor-stator combinations, and multi-stage compressors or turbines. The steps from single to multi-element airfoil design, and from two to three dimensions are both substantial.

Such a large range of possibilities cannot be tackled all at once. An immediately promising direction is to move toward an initial goal of optimization of supersonic wing and wing-body configurations, perhaps using the conical flow assumption as an intermediate step to moderate the computational costs while gaining experience and insight. Since these flows can be modelled by both the potential flow and the Euler equations, this also offers a convenient setting for an investigation of the problems associated with using the more complex Euler model. Among the key issues in this investigation are the choice of proper boundary conditions, discretization of both the flow and the adjoint equations, and accelerated solution methods.

References

1. J. C. Doyle, K. Glover, P. P. Khargonekar and B. A. Francis, "State-Space Solutions to Standard H_2 and H_∞ Control Problems," IEEE Trans. Automatic Control, 34, 1989, pp. 831-847.

2. R. A. Hyde and K. Glover, "VSTOL Aircraft Flight Control System Design Using H_∞ Controllers and a Switching Strategy," Proc. 29th IEEE Conference on Decision and Control, Honolulu, December 1990, pp. 2975-2980.

3. I. Kaminer and P. P. Khargonekar, Design of the "Flare Control Law for Longitudinal Autopilot Using H_∞ Synthesis," Proc. 29th IEEE Conference on on Decision and Control, Honolulu, December 1990, pp. 2981-2986.

4. J. L. Lions, "Optimal Control of Systems Governed by Partial Differential Equations," translated by S. K. Mitter, Springer Verlag, New York, 1981.

5. A. Jameson, "Successes and Challenges in Computational Aerodynamics," AIAA Paper 87-1184CP, Proc. 8th Computational Fluid Dynamics Conference, Honolulu, June 1987, pp. 1-35.

6. A. Jameson, "Computational Aerodynamics for Aircraft Design," Science, 245, 2989, pp. 3611-371.

7. A. Jameson, T. J. Baker and N. P. Weatherill, "Calculation of Inviscid Transonic Flow Over a Complete Aircraft," AIAA Paper 86-0103, 24th Aerospace Sciences Meeting, Reno, January 1986.

8. J. C. Vassberg and K. B. Dailey, "AIRPLANE, Experience, Benchmarks and Improvements," AIAA Paper 90-2998, 8th Applied Aerodynamics Conference, Portland, August 1990.

9. A. H. Epstein, E. M. Greitzer, G. R. Guerette and L. Valavani, "Active Compressor Stabilization," Proc. 29th IEEE Conference on Decision and Control, Honolulu, November 1990.

10. A. Jameson, "Aerodynamic Design Via Control Theory", ICASE Report 88-64, 1988, J. of Scientific Computing, 3, 1988, pp 233-260.

11. A. Jameson, "Automatic Design of Transonic Airfoils to Reduce the Schock Induced Pressure Drag," Proc 31st Israel Annual Conference on Aviation and Aerodynamics, February 1990, pp. 5-17.

12. A. Jameson, "Control Theory Applications for Optimum Design of Aerodynamic Shapes," Proc. 29th IEEE Conference on Decision and Control. Honolulu, December 1990, pp. 176-179.

13. M. J. Lighthill, "A New Method of Two Dimensional Aerodynamic Design," ARC R. and M. 2112, 1945.

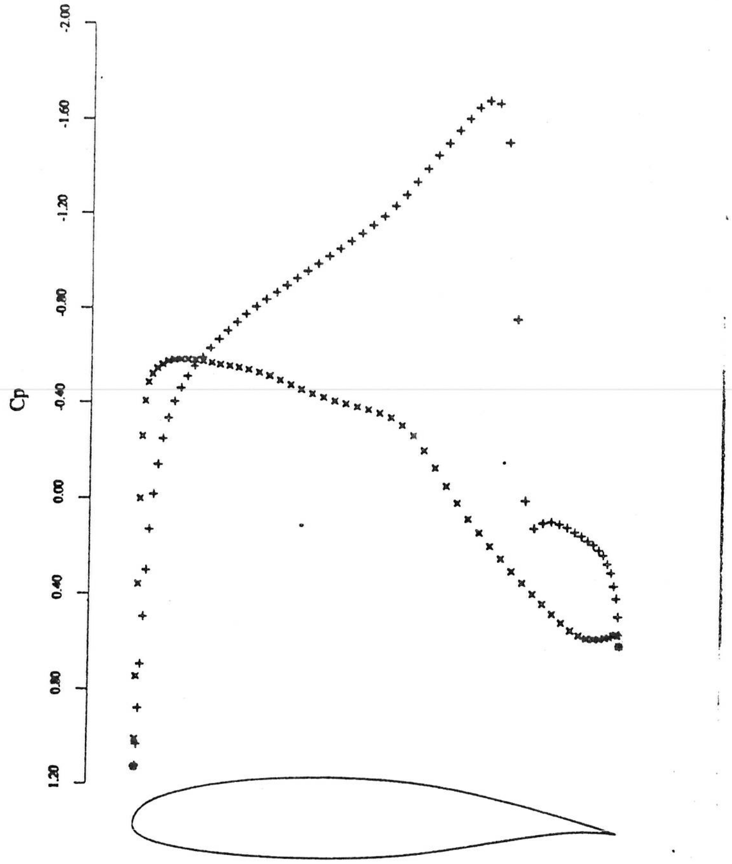
14. P. A. Henne, "An Inverse Transonic Wing Design Method," AIAA Paper 80-0330, 18th Aerospace Sciences Meeting, 1980.

15. M. J. Siclari, "Computation of Nonlinear Supersonic Potential Flow Over Three-Dimensional Surfaces," J. of Aircraft, 20, 1983, p. 462.

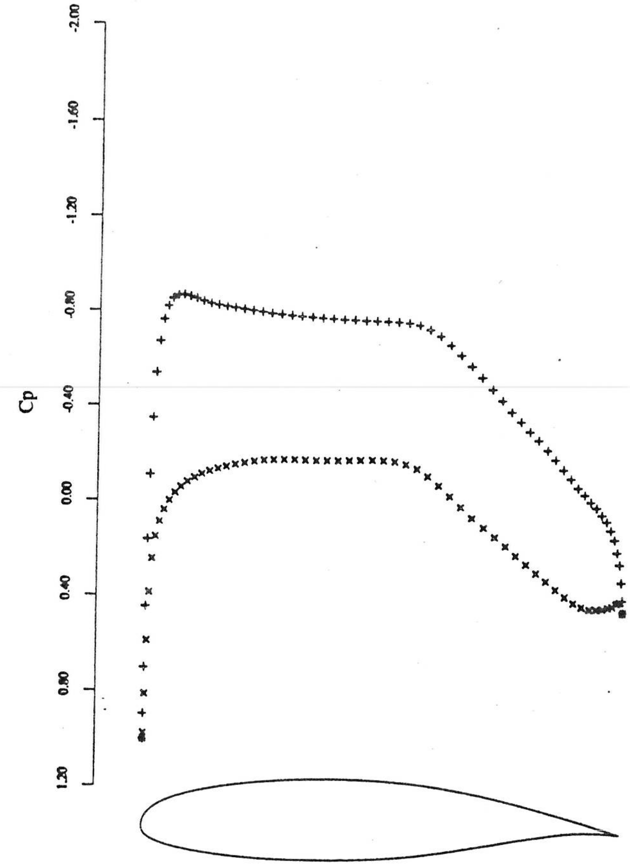
16. M. J. Siclari and P. Guidice, "A Hybrid Finite Volume Approach to Euler Solutions for Supersonic Flows," AIAA Paper 88-0225, 26th Aerospace Sciences Meeting, Reno, January 1988.

17. R. T. Jones, "The Minimum Drag of Thin Wings in Frictionless Flow," J. Aeronautical Sciences, 18, 1951, pp. 75-81.

18. B. Grossman, "Numerical Procedure for the Computation of Irrotational Conical Flows," AIAA J., 17, 1979, p. 828.



GAWX AIRFOIL
MACH 0.720 ALPHA -2.526
CL 0.5982 CD 0.0191 CM -0.2381
GRID 128X32 NCYC 0 RES0.119E-04



GAWX AIRFOIL
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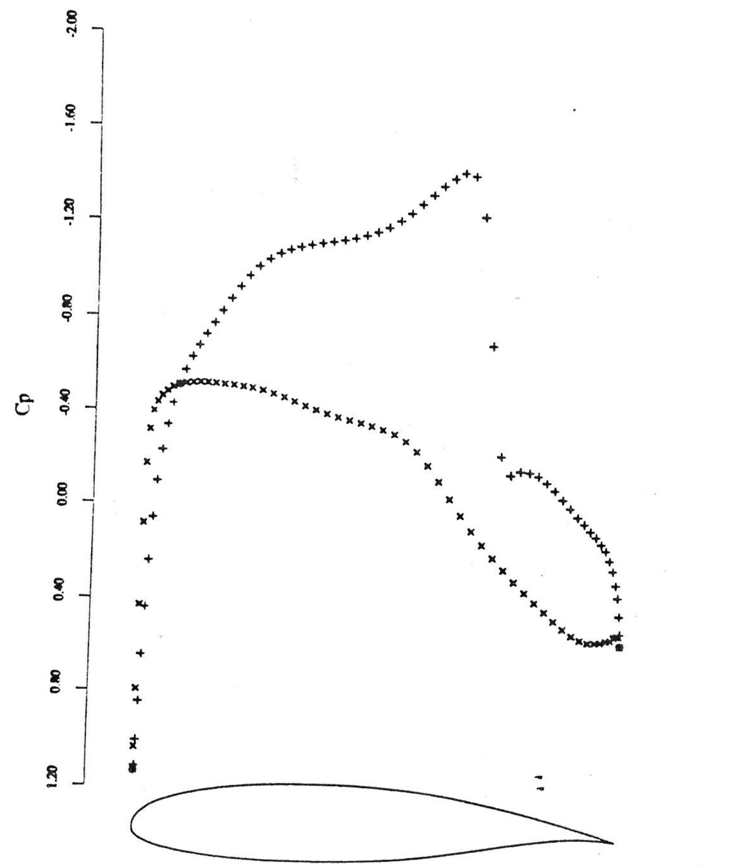
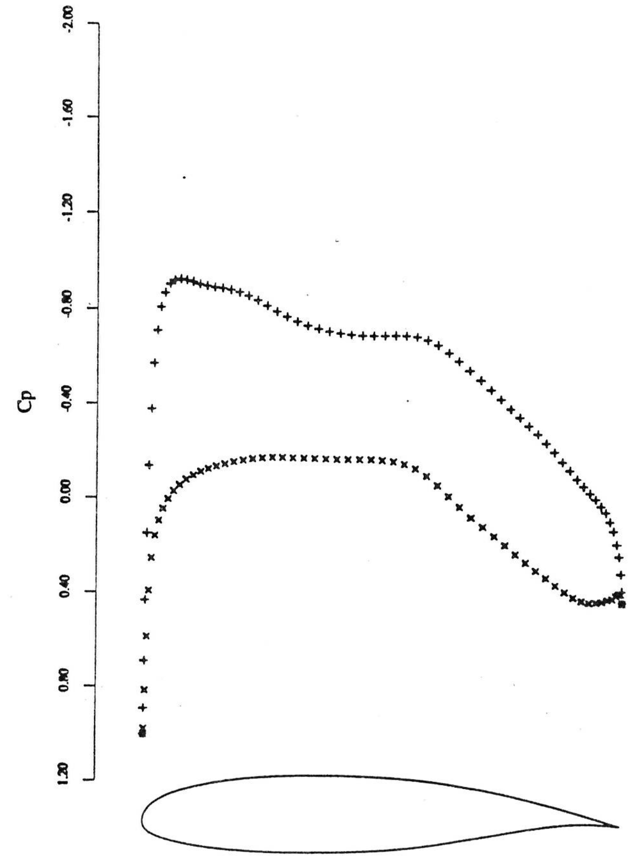
CL 0.5998 CD -0.0001 CM -0.1288
GRID 128X32 NCYC 0 RES0.488E-11

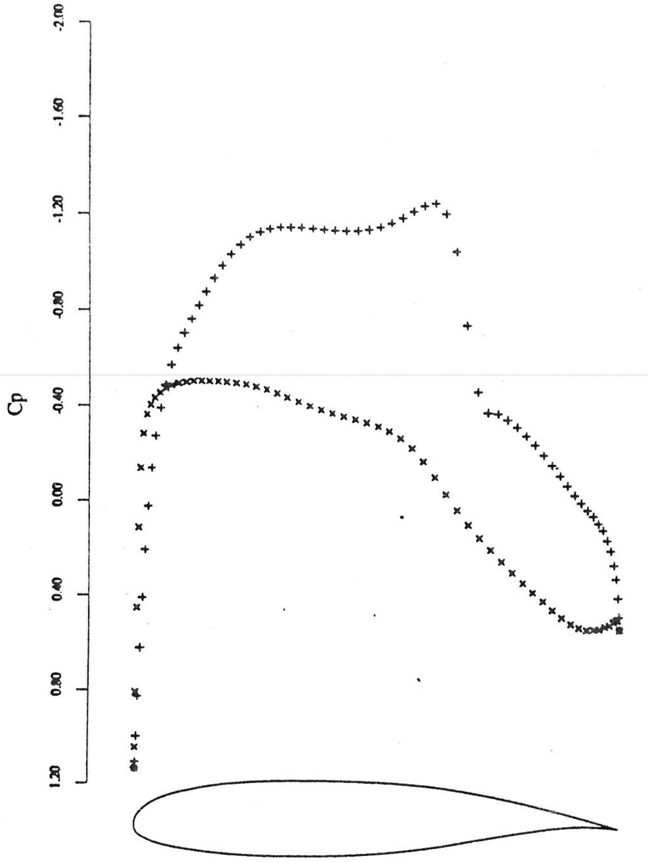


GAWX AIRFOIL
MACH 0.720 ALPHA -2.110
CL 0.5999 CD 0.0057 CM -0.2006
GRID 128X32 NCYC 2 RES0.736E-06

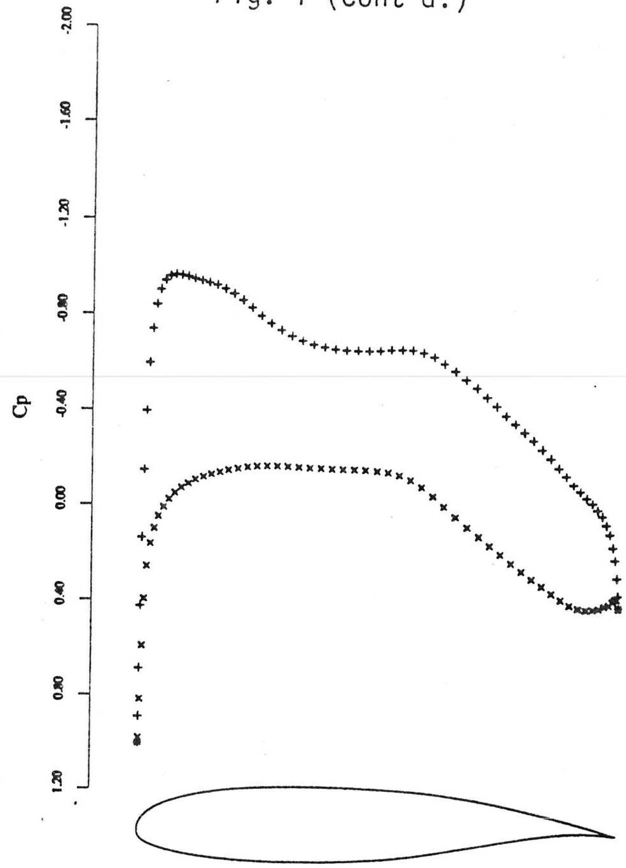


GAWX AIRFOIL
MACH 0.200 ALPHA 0.188
CL 0.5998 CD -0.0001 CM -0.1246
GRID 128X32 NCYC 2 RES0.590E-11

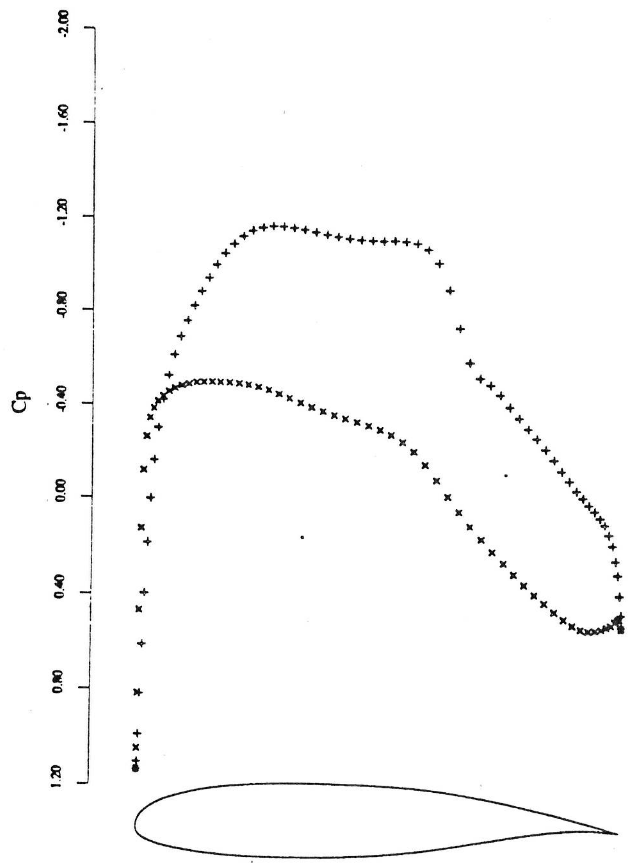
Fig. 1 (Cont'd.)



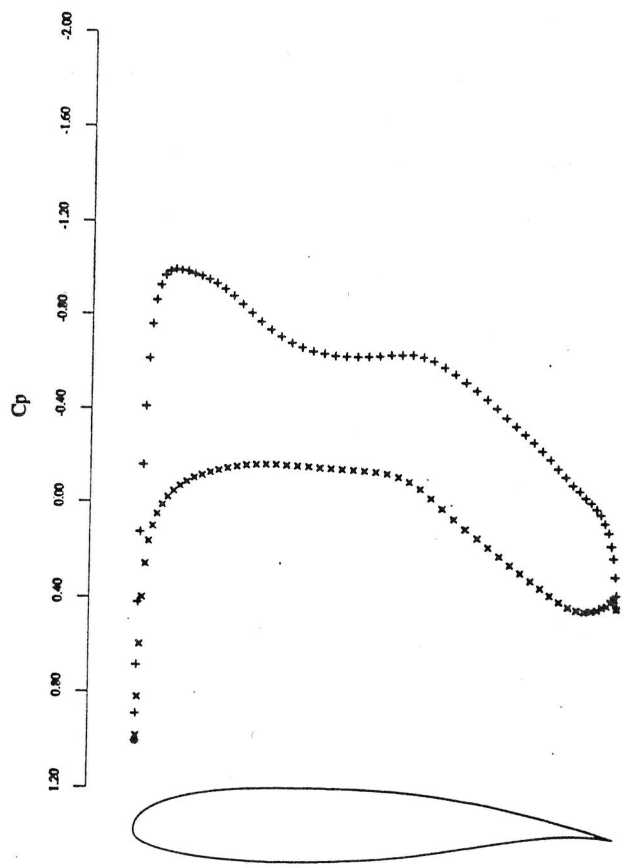
GAWX AIRFOIL
MACH 0.720 ALPHA -1.952
CL 0.6001 CD 0.0015 CM -0.1874
GRID 128X32 NCYC 4 RES0.820E-06



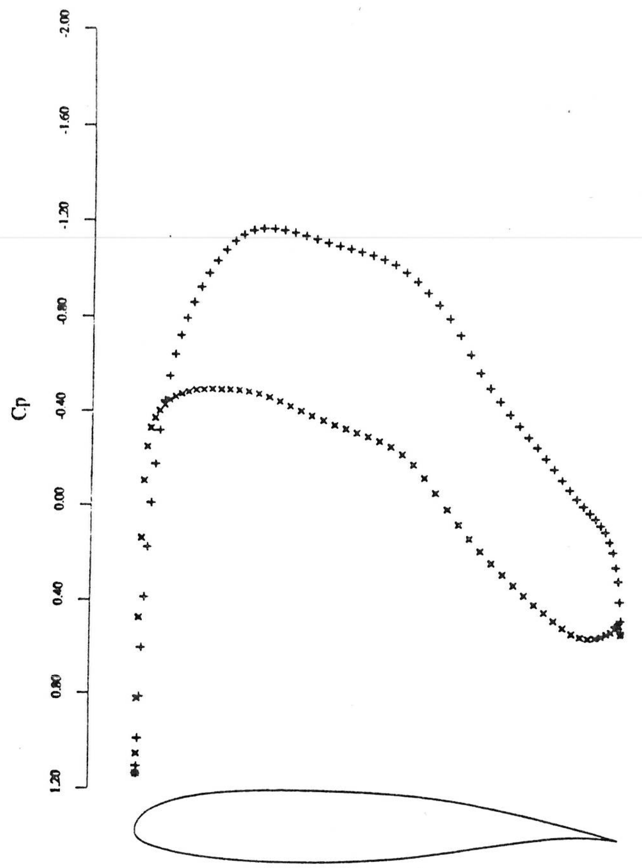
GAWX AIRFOIL
MACH 0.200 ALPHA 0.201
CL 0.5998 CD -0.0001 CM -0.1237
GRID 128X32 NCYC 4 RES0.631E-11



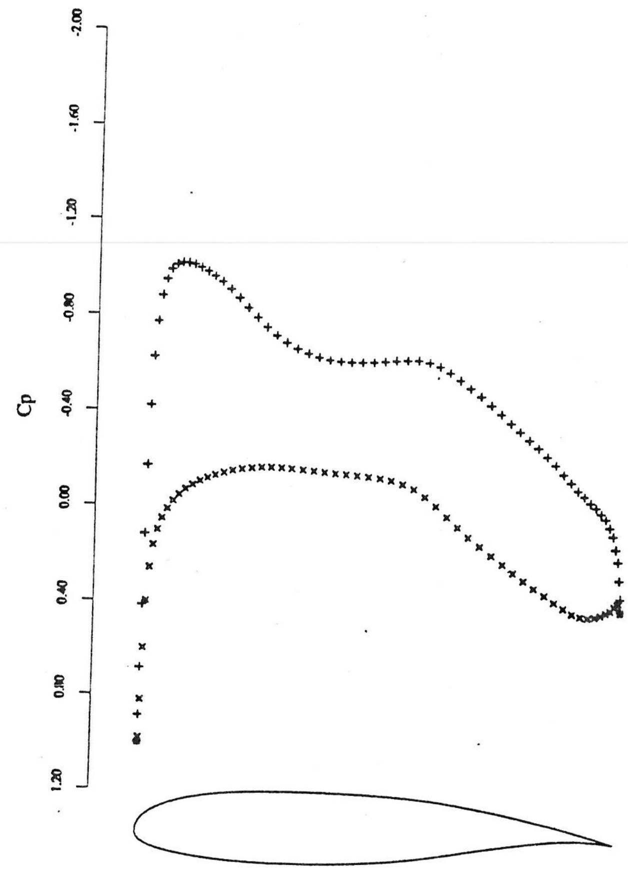
GAWX AIRFOIL
MACH 0.720 ALPHA -1.875
CL 0.6000 CD 0.0003 CM -0.1819
GRID 128X32 NCYC 6 RES0.390E-06



GAWX AIRFOIL
MACH 0.200 ALPHA 0.211
CL 0.5998 CD -0.0001 CM -0.1236
GRID 128X32 NCYC 6 RES0.672E-11



GAWX AIRFOIL
MACH 0.720 ALPHA -1.834
CL 0.5999 CD 0.0001 CM -0.1793
GRID 128X32 NCYC 8 RES0.923E-08



GAWX AIRFOIL
MACH 0.200 ALPHA 0.218
CL 0.5998 CD -0.0001 CM -0.1236
GRID 128X32 NCYC 8 RES0.700E-11

Aug 1, 1991-June 30, 1994

	Year 1	Year 2	Year 3
Salaries			
Prof. Antony Jameson, P.I.			
Academic Year 10%	11510	12085	12690
1 Summer Month	12788	13428	14100
Post-Doctoral Associate			
Calendar Year 40%	12000	12600	13230
Total Salaries	36298	38113	40020
Benefits @ 30.7, 31.5 32.3 %	11143	12006	12926
Total Salaries and Benefits	47441	50119	52946
Graduate Student (1) Pre-General			
Academic Year Stipend	10000	10500	11000
Summer Stipend	4200	4400	4600
Materials	1500	1800	2000
Travel	4000	4000	4000
Total Direct Costs	67141	70819	74546
Indirect Costs @ 67%	44985	47448	49946
Tuition	8290	8845	.9440
Equipment, Convex Computer	30000	25000	20000
Total Costs to Sponsor	150416	152112	153933