High-Order Methods for Turbulent Flow Simulations on Deforming Domains

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Joint work with B. Froehle, L. Wang, S. Kanner, M. Fortunato, M. Zahr, D. J. Willis, S. Govindjee, J. Peraire, X. Roca, G. Dahlbacka

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Berkeley Lab

High-Order Discontinuous Galerkin Simulations

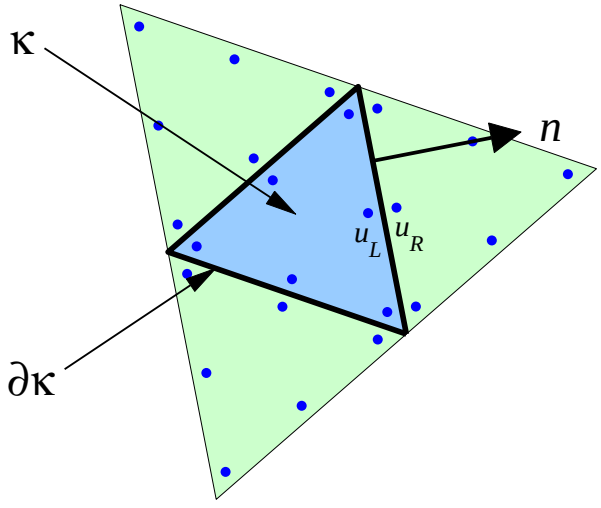
- Discontinuous Galerkin (DG) methods have desirable properties:

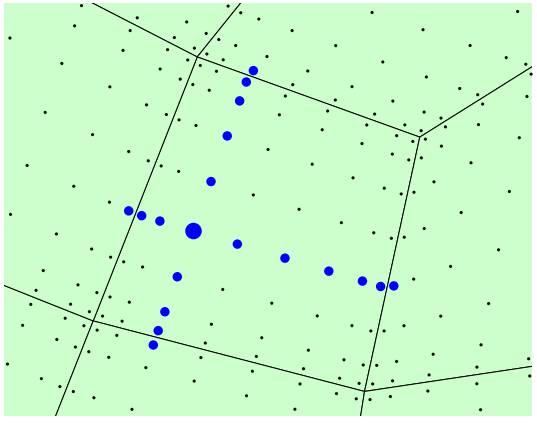
	FVM	FDM	FEM	DG
1) High-order/Low dispersion	✗	✓	✓	✓
2) Complex geometries	✓	✗	✓	✓
3) Stability for conservation laws	✓	✓	✗	✓

- However, several problems to resolve:
  - High CPU/memory requirements (compared to FVM or H-O FDM)
  - Robustness issues, low tolerance to under-resolved features
  - High-order geometry representation and mesh generation
- *Need to make DG competitive for real-world problems*

The 3DG Software Package

- Standard nodal high-order DG discretization for conservation laws
- Curved, fully unstructured meshes of tetrahedra/hexahedra
- General multiphysics formulation – same code used for fluid/structure, for DG/CG, etc, with modular interfaces
- Fully implicit formulation, sparse block formats for Jacobians
- Highly sparse “Line-DG” formulation for quad/hex elements

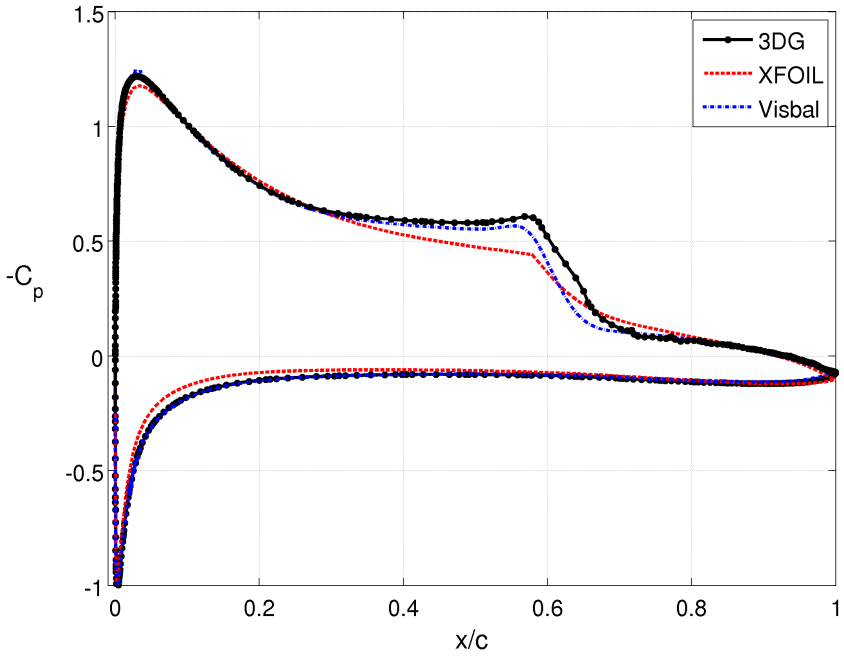


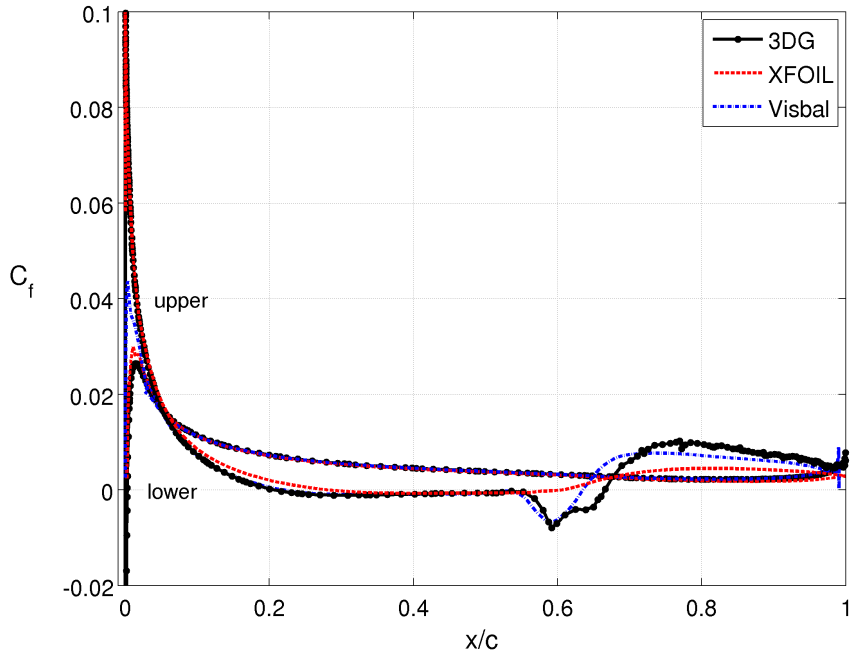


- Efficient parallel solvers with matrix-based decompositions– nearly perfect speedup to 10,000’s of cores for time-accurate problems

Example: ILES at *Re* = 60,000

- Implicit LES for flow past airfoil [Uranga *et al* 2009, 2011]
- Separation and transition well captured
- Good agreement with XFoil and previously published ILES





Average pressure and skin friction coefficients

Computational Domain

2D and 3D NACA0012 Airfoil for $0^\circ \leq \alpha \leq 90^\circ$

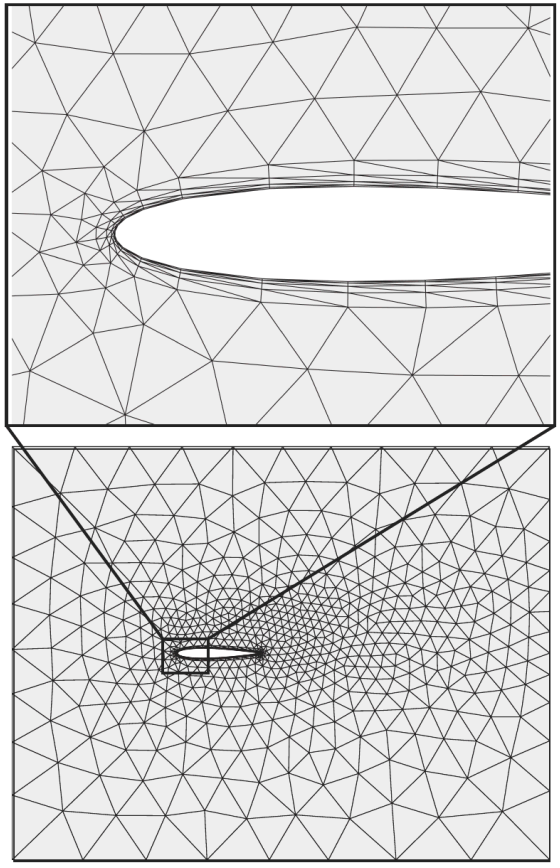


Figure: Mesh 0 (2D)

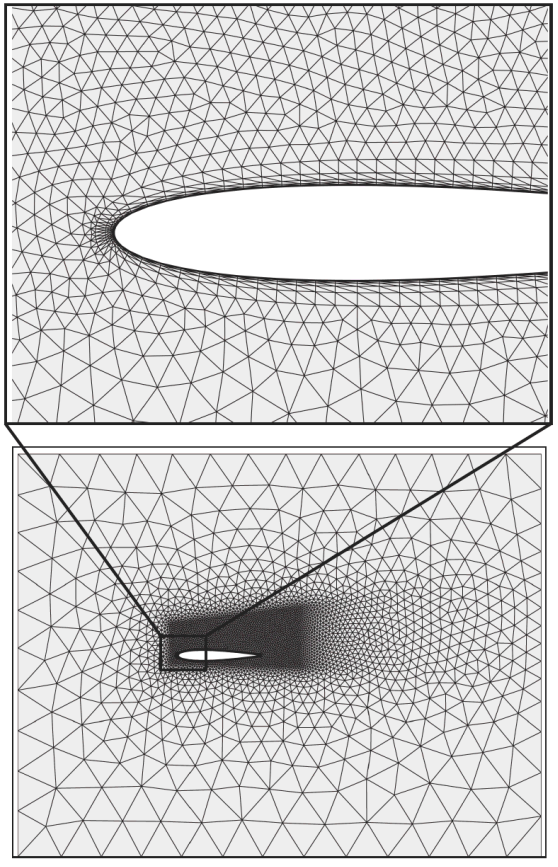


Figure: Mesh 2 (2D)

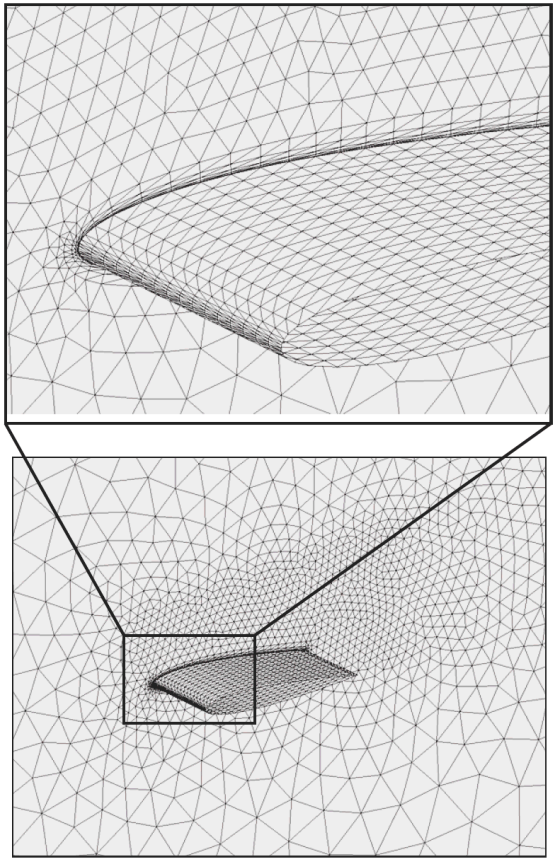


Figure: Mesh 1 (3D)

Flow Structure, single NACA Airfoil at Re=5.3k

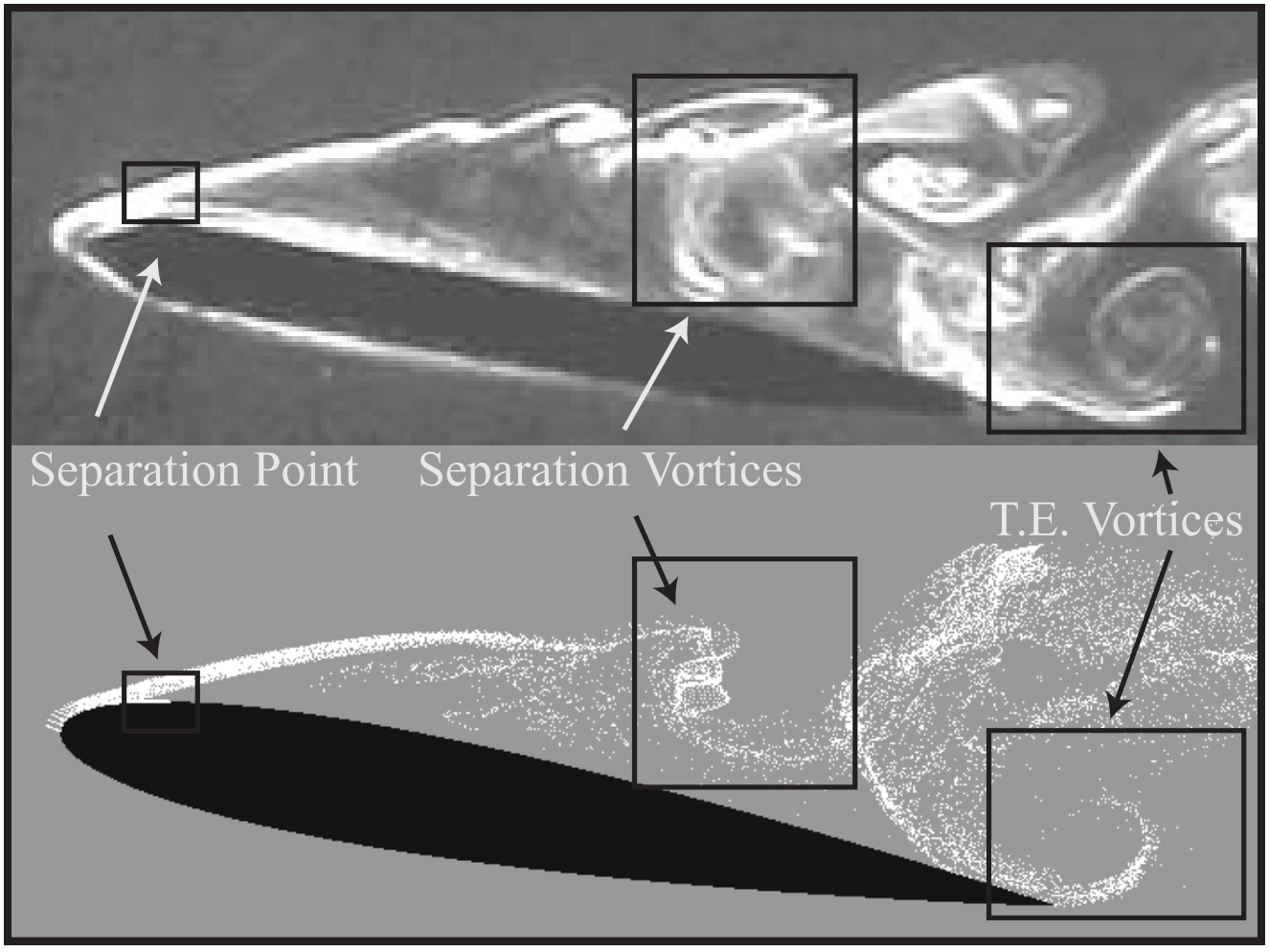


Figure: Comparison of flow structure to LIF visualization

Forces, single NACA Airfoil at Low Re

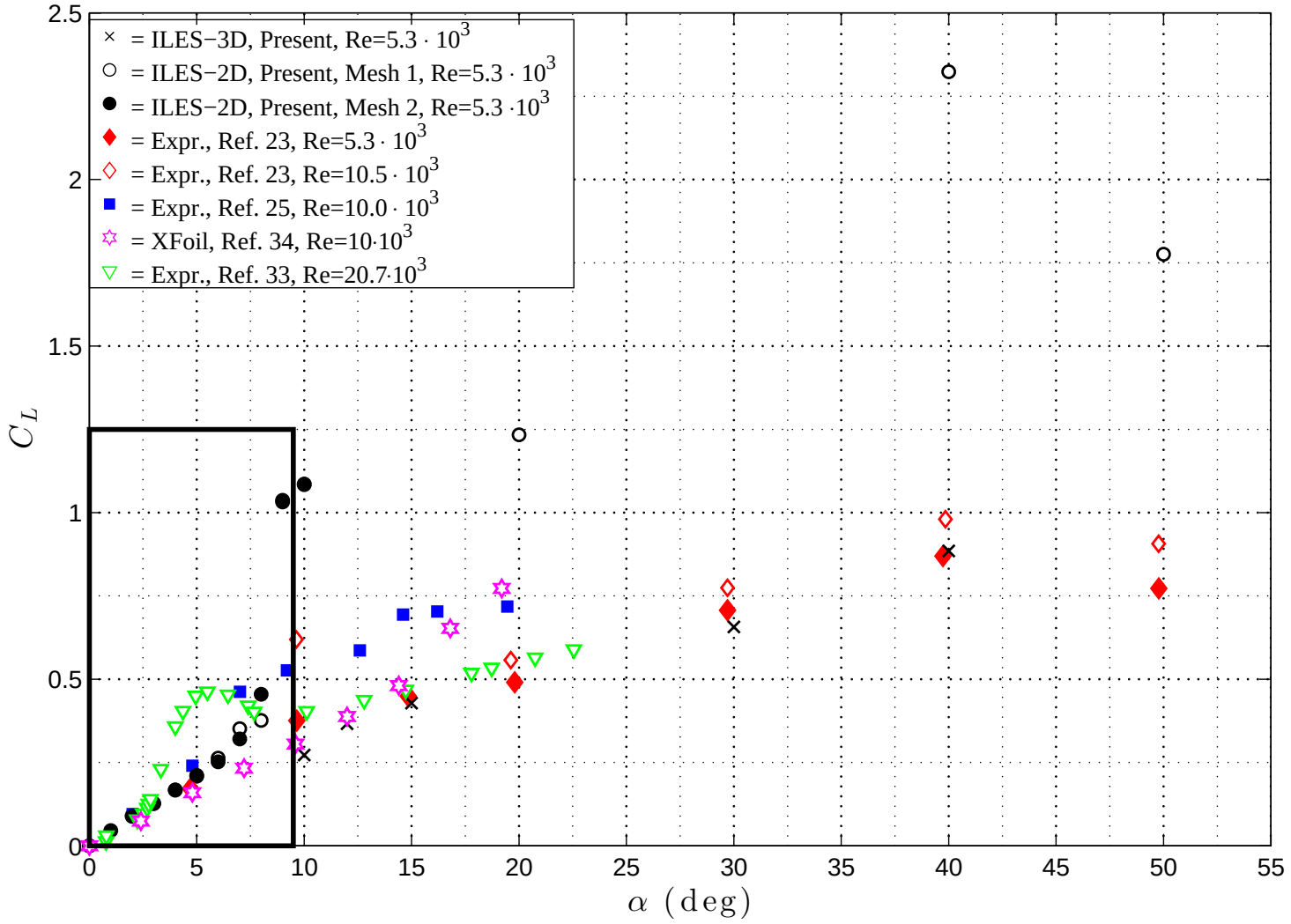


Figure: Lift Coefficient for $0° \leq \alpha \leq 55°$ with $5,000 \leq Re \leq 21,000$

Forces, single NACA Airfoil at Re=40k

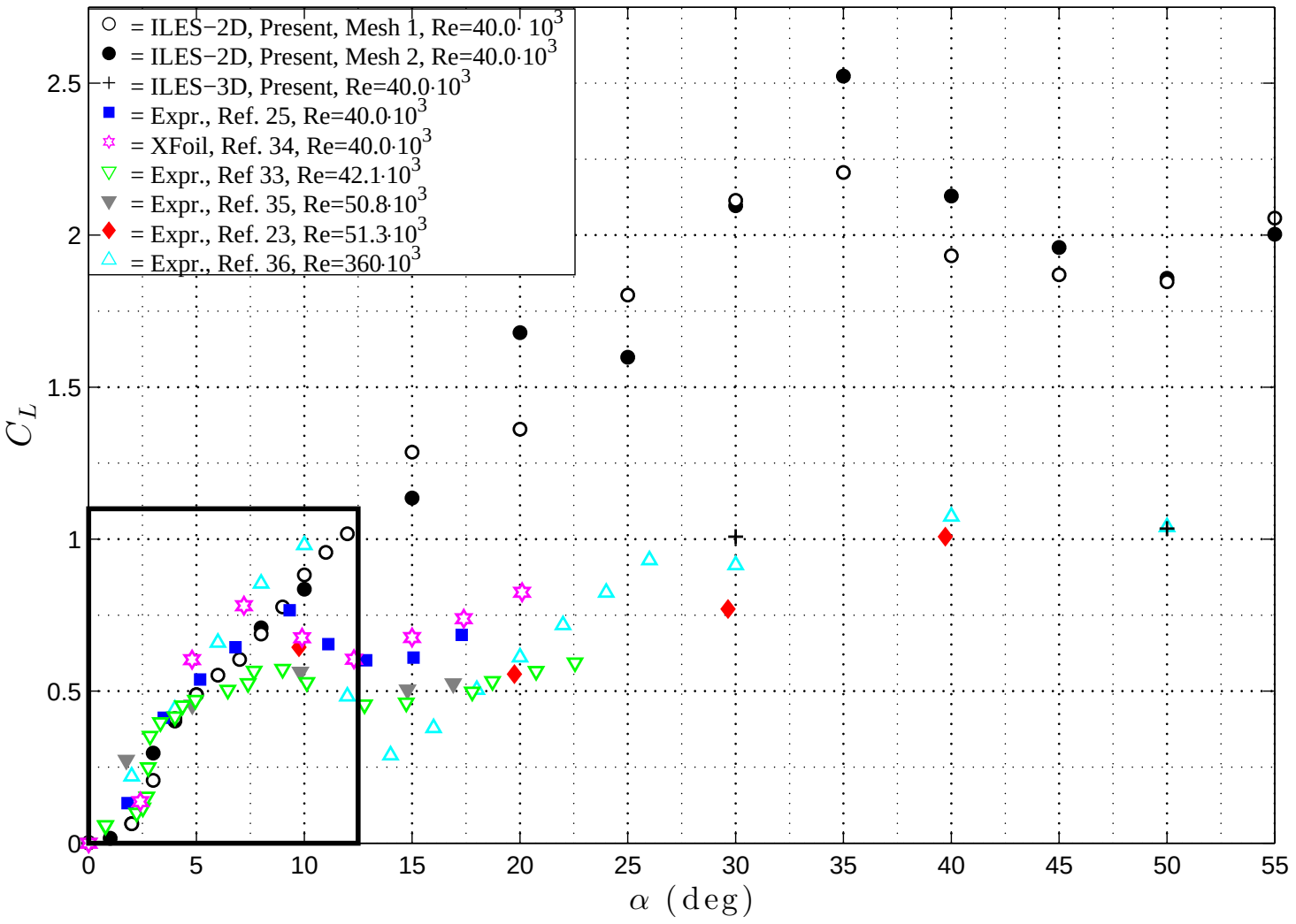


Figure: Lift Coefficient for $0° \leq \alpha \leq 55°$ with $Re \geq 40,000$

Frequency content of forces

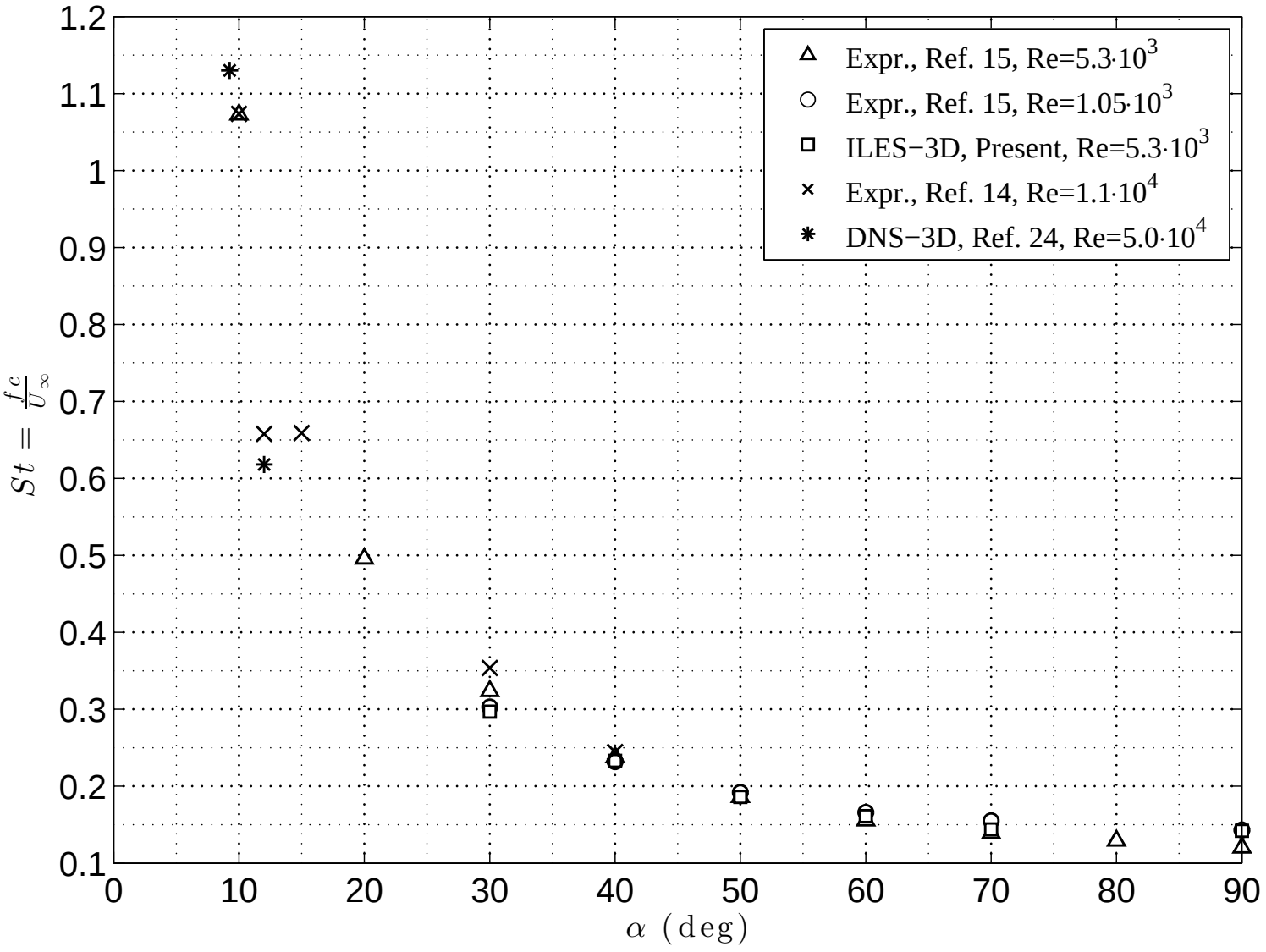
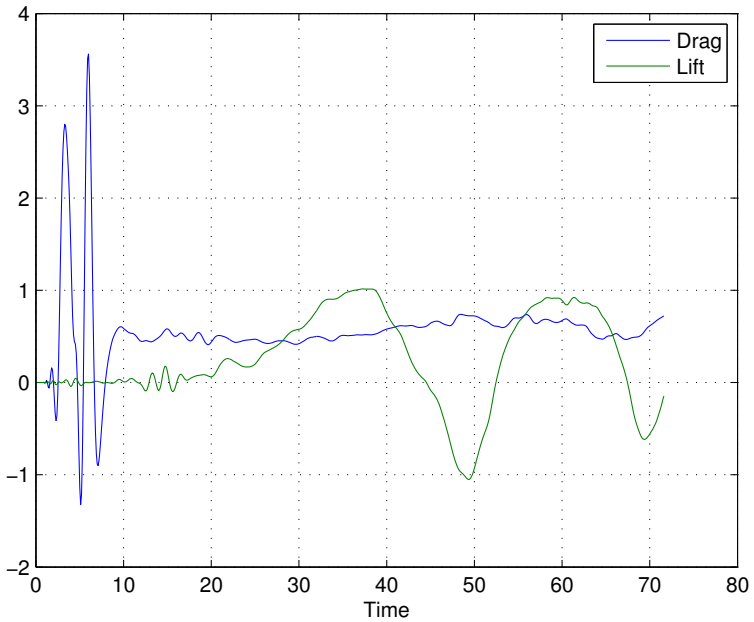
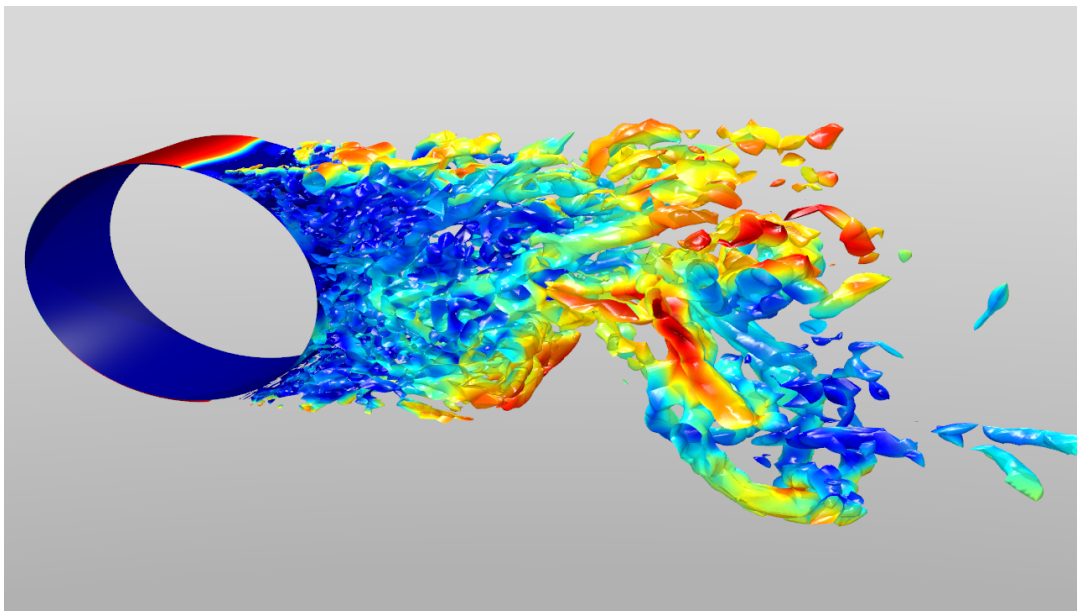


Figure: Frequency content of lift force at Re= 5.3x10^3 (Expr from Alam, 2011).

Cylinder – Delayed Detached Eddy Simulation

- DES/DDES [Spalart 1997,2006]: Hybrid RANS/LES model for problems with significant flow separation
- Edge of boundary layer moving; stabilize by continuous AV
- Model problem: Flow over cylinder, $\mathrm{Re} = 1.5 \cdot 10^6$
- Left: Force coefficients C_D, C_L, Right: Q-criterion isosurfaces





ALE Formulation for Deforming Domains

- Use mapping-based ALE formulation for moving domains

  [Visbal,Gaitonde 2002], [Persson,Bonet,Peraire 2009]
- Map from reference domain V to physical deformable domain $v(t)$
- Introduce the *mapping deformation gradient* $\boldsymbol{G} = \boldsymbol{\nabla}_X \boldsymbol{\mathcal{G}}$ and the *mapping velocity* $\boldsymbol{v}_X = \frac{\partial \boldsymbol{\mathcal{G}}}{\partial t}\big|_X$, and set $g = \det(\boldsymbol{G})$
- For numerically computed grid motions, compute *stage consistent* velocities by imposing

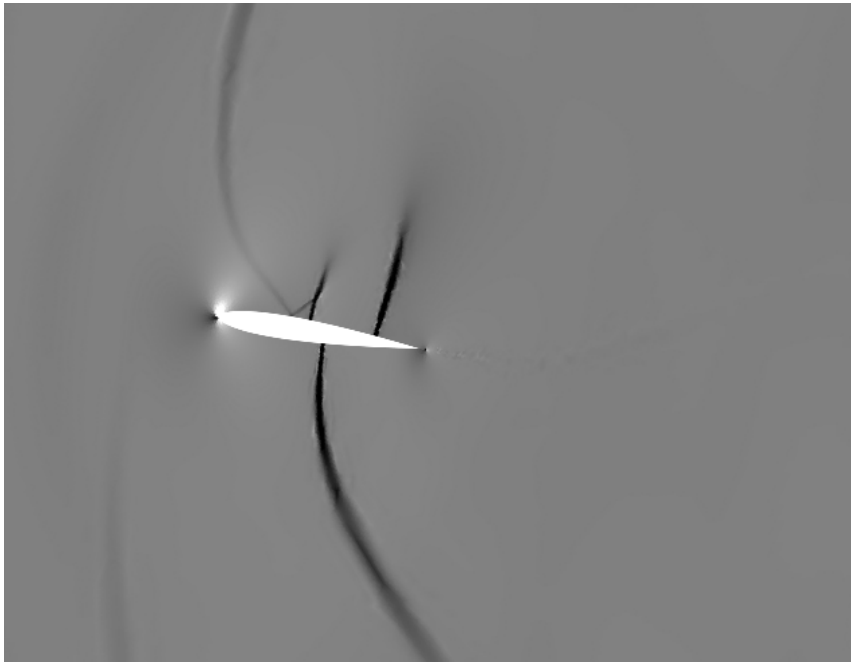
$$\boldsymbol{x}_i = \boldsymbol{x}_0 + \Delta t \sum_{j=1}^{s} a_{ij} \boldsymbol{\nu}_j \quad \Longrightarrow \quad \boldsymbol{\nu}_i = \sum_{j=1}^{s} (A^{-1})_{ij} \frac{\boldsymbol{x}_j - \boldsymbol{x_0}}{\Delta t}, \quad i = 1, \ldots, s,$$

  where A is the implicit Runge-Kutta Butcher tableaux

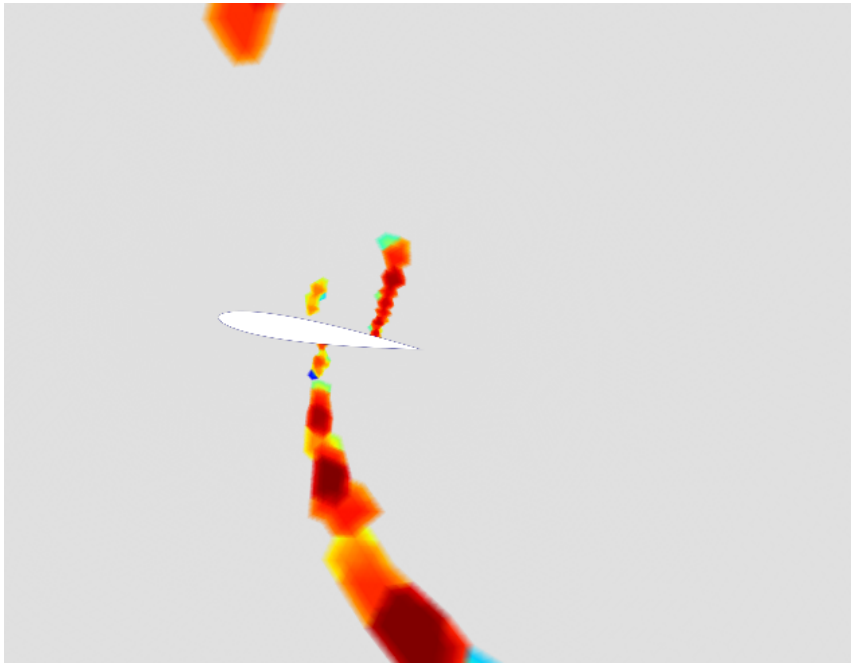
  [Froehle & Persson, in review]
- Transform equations to account for the motion

Pitching wing in transonic flow

- ALE formulation for the Euler equations combined with sensor-based artificial diffusion
- Freestream $\mathrm{Ma} = 0.6$, AoA harmonic with amplitude = 30°
- Implicit solver, re-used Jacobians



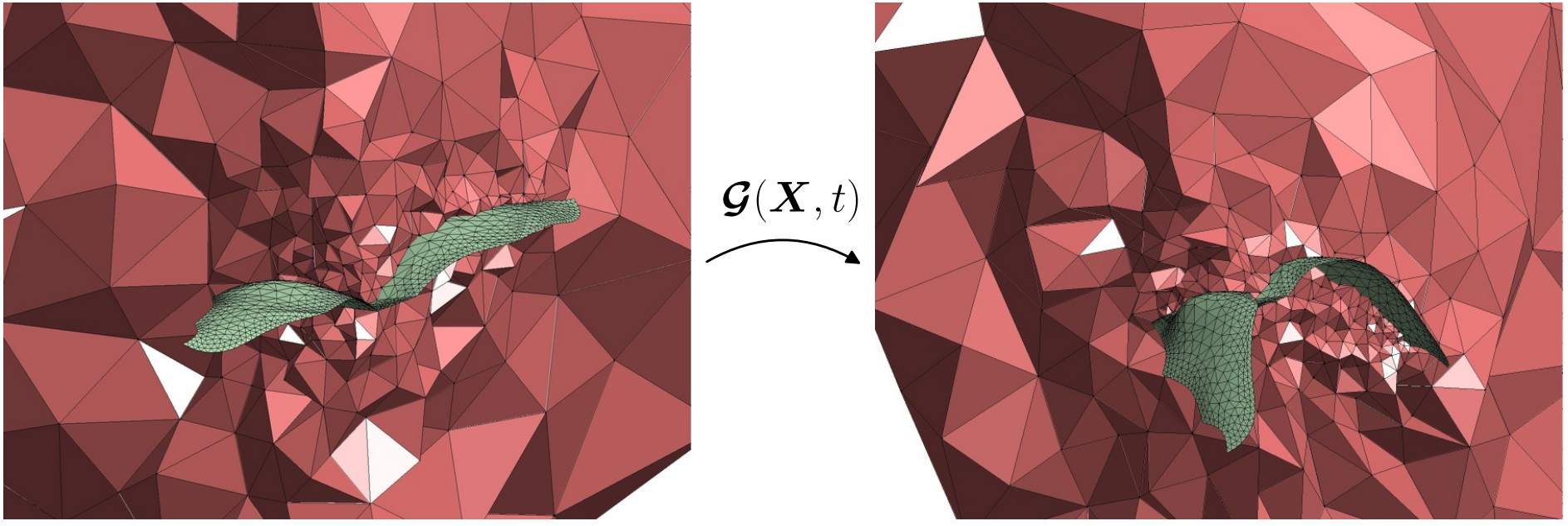
x-gradient of density



Artificial viscosity

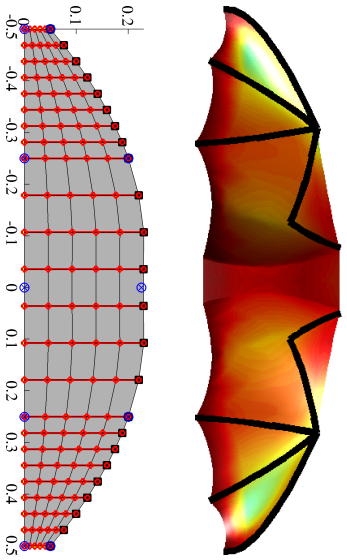
Nonlinear Elasticity for Deforming Domains

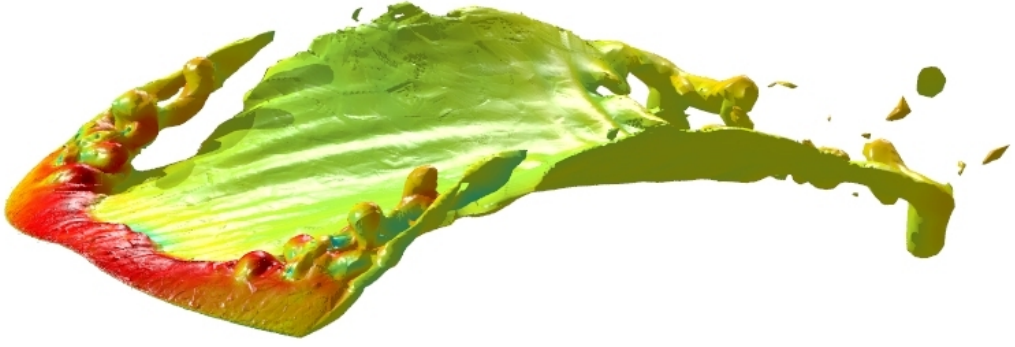
- Non-linear solid mechanics approach used to deform meshes
- High-order ALE methods require a smooth mapping $\boldsymbol{\mathcal{G}}(\boldsymbol{X}, t)$ such that the elements are aligned with the moving boundaries
- Example: Flapping bat wings, wing motion from experimental data
- A reference mesh (left) is deformed elastically (right)

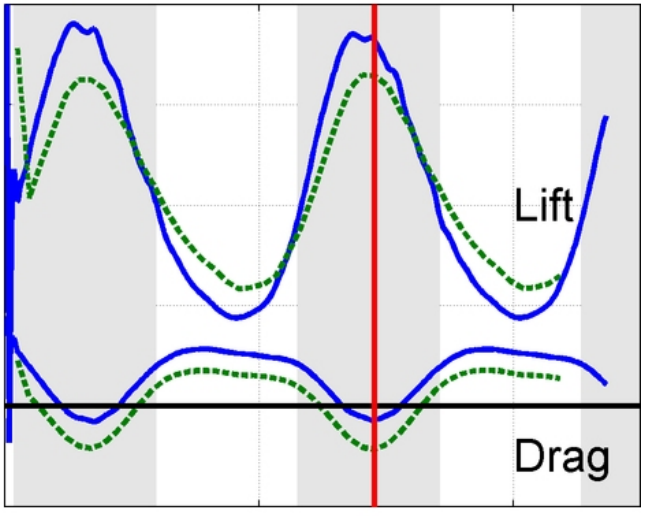


Optimal Flapping Kinematics

- **Goal:** To design and analyze an effective flapping wing shape for cruising flight
- A *multi-fidelity approach* with a range of simulation tools [Willis & Persson 2011, 2014]







Vertical Axis Wind Turbines

- Recent interest in vertical axis wind turbines (VAWT):
  - 2D airfoils, easy to manufacture, supportable at both ends
  - Omnidirectional (good in gusty, low wind, e.g. close to ground)
  - Lower blade speeds – lower noise and impact
  - Can be packed close together in wind farms
- Numerical simulations can help overcome remaining challenges:
  - Lower theoretical (and practical) efficiency than HAWTs
  - Sensitive to design conditions
  - Structural problems, fatigue and catastrophic failure

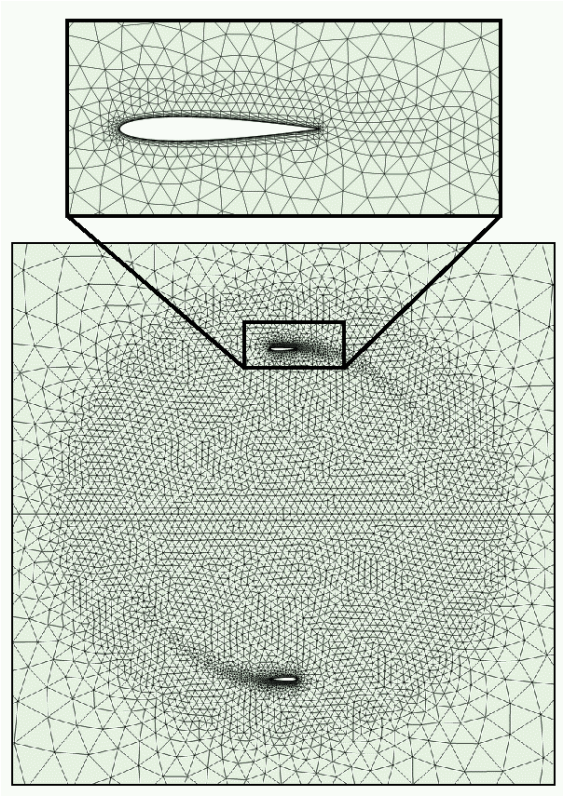
(3kW unit, G. Dahlbacka, LBNL)

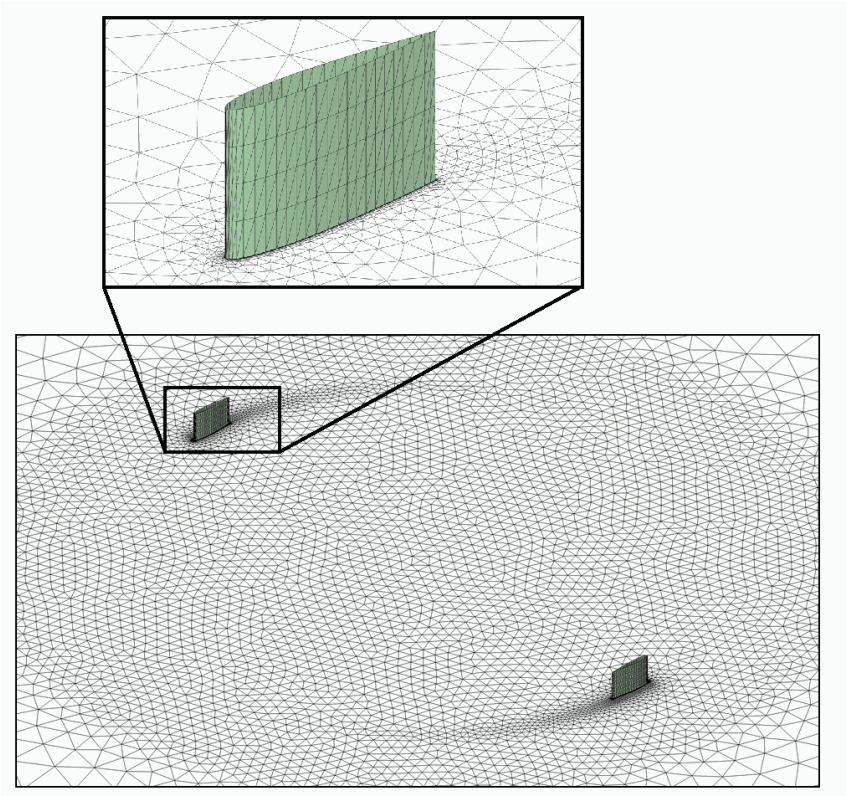
VAWT – Mathematical Model and Discretization

- Solve the Navier-Stokes equations in a rotating frame:

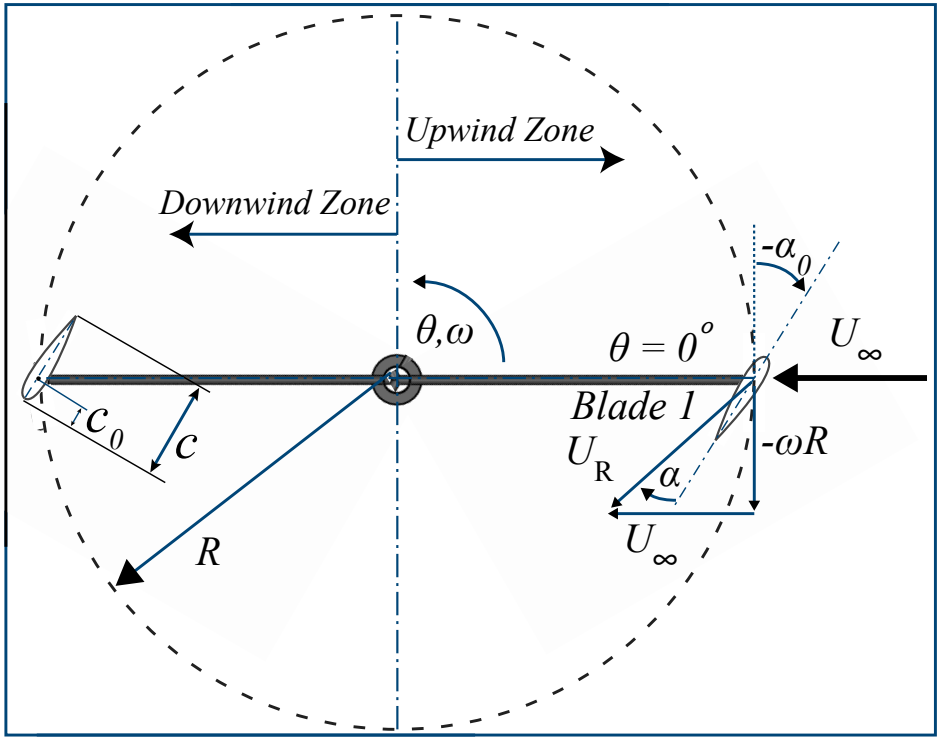
$$\mathcal{G}(X, Y, t) = \begin{bmatrix} \cos \omega t & -\sin \omega t \\ \sin \omega t & \cos \omega t \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

- Hybrid boundary layer/unstructured mesh, element degree $p = 3$

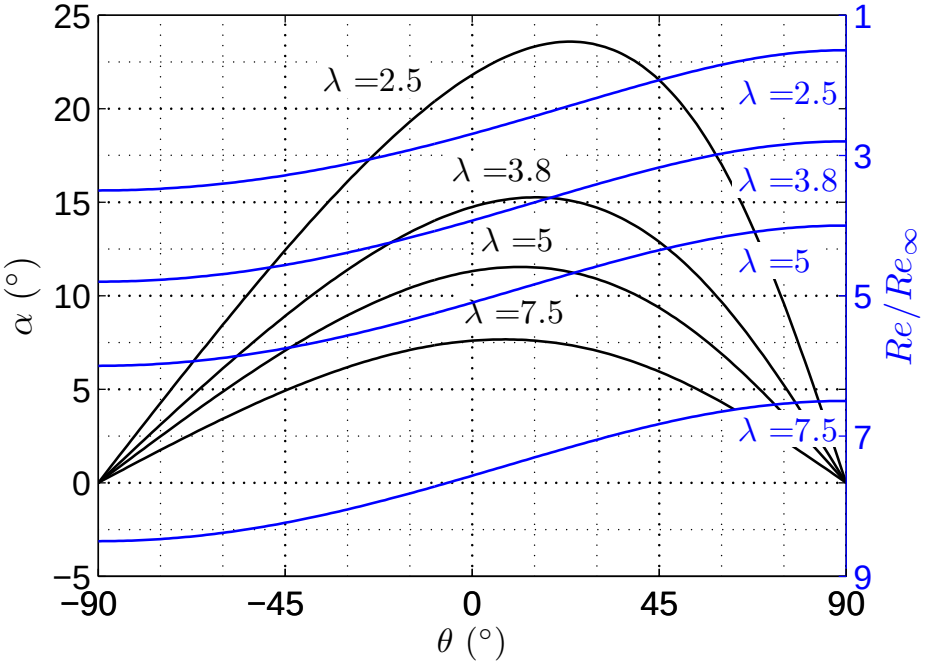




Range of angle-of-attack (α) based on TSR (λ)



Bird's-eye-view of 2-bladed VAWT

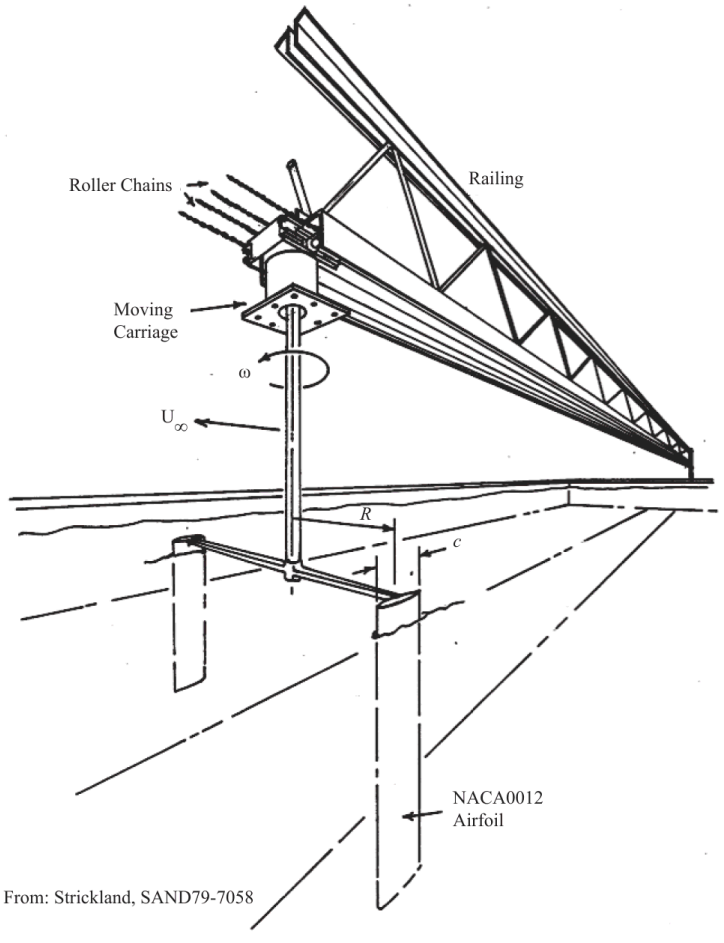


Variation of angle-of-attack at various TSR as a function of azimuthal angle

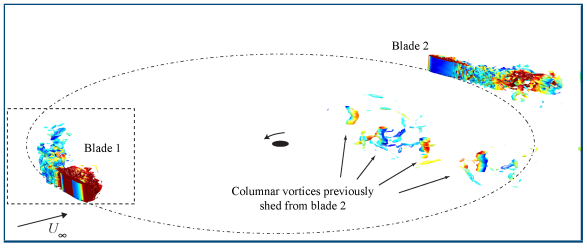
$$\lambda = \frac{\omega R}{U_\infty}$$

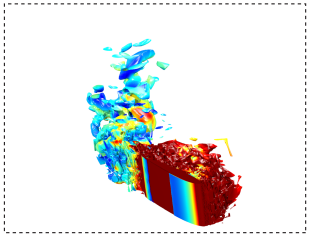
Sandia National Laboratories' Low Re VAWT

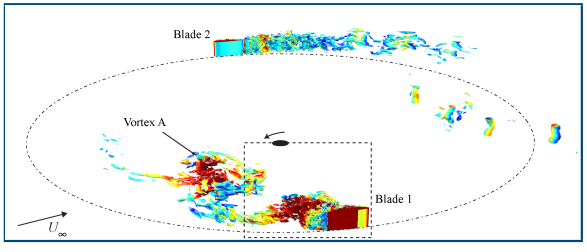
Sandia National Labs tow tank experiment from 1979

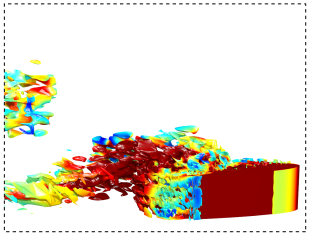


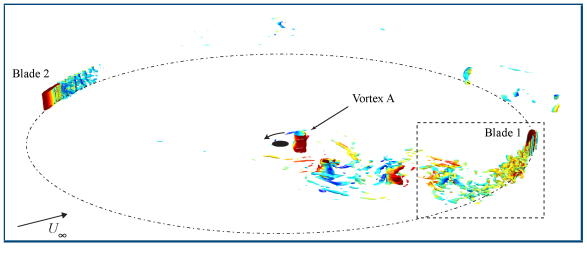
From: Strickland, SAND79-7058

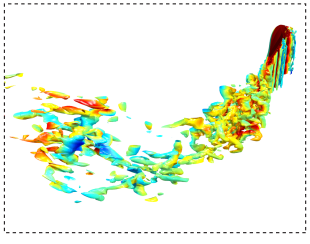






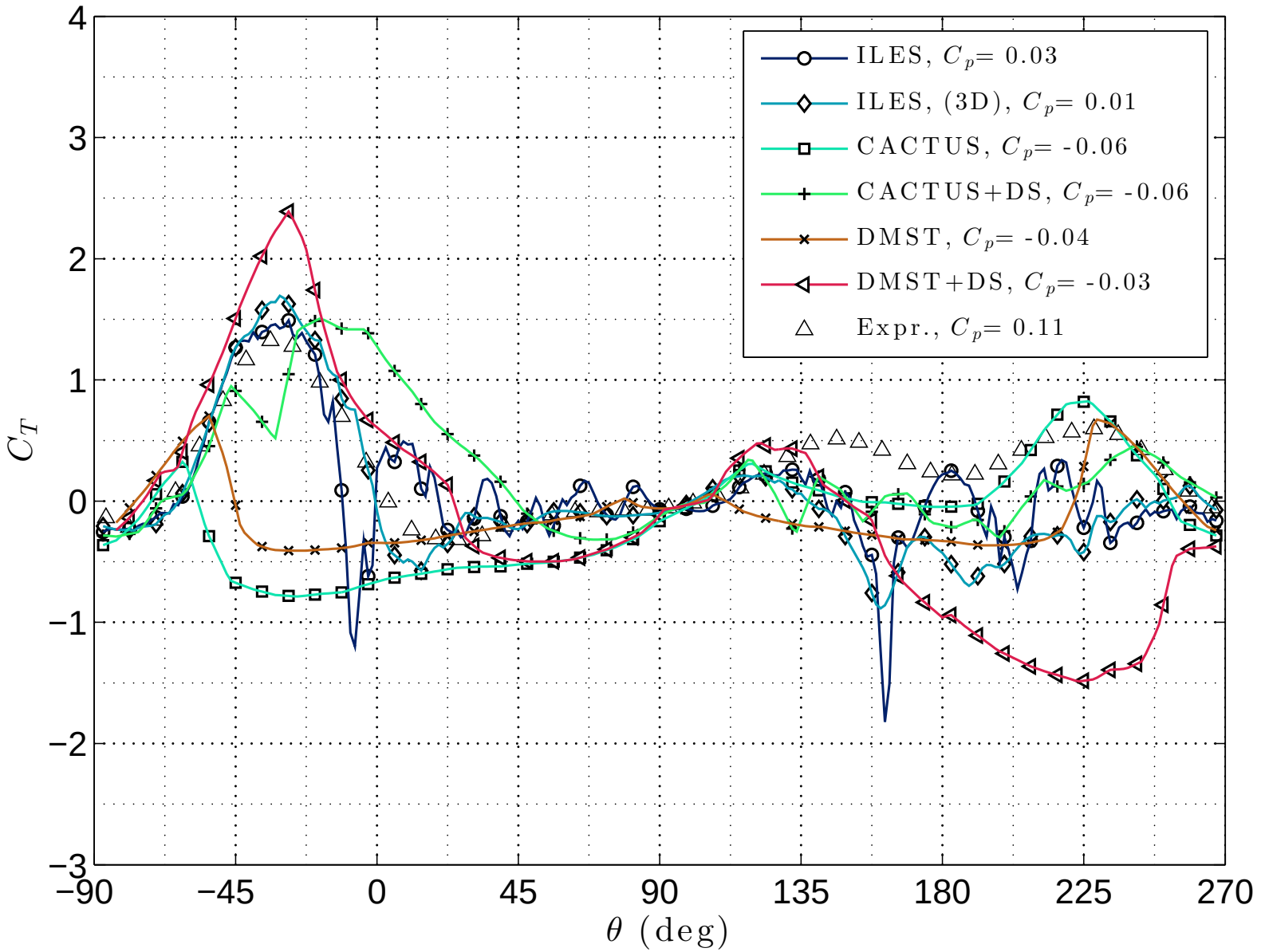






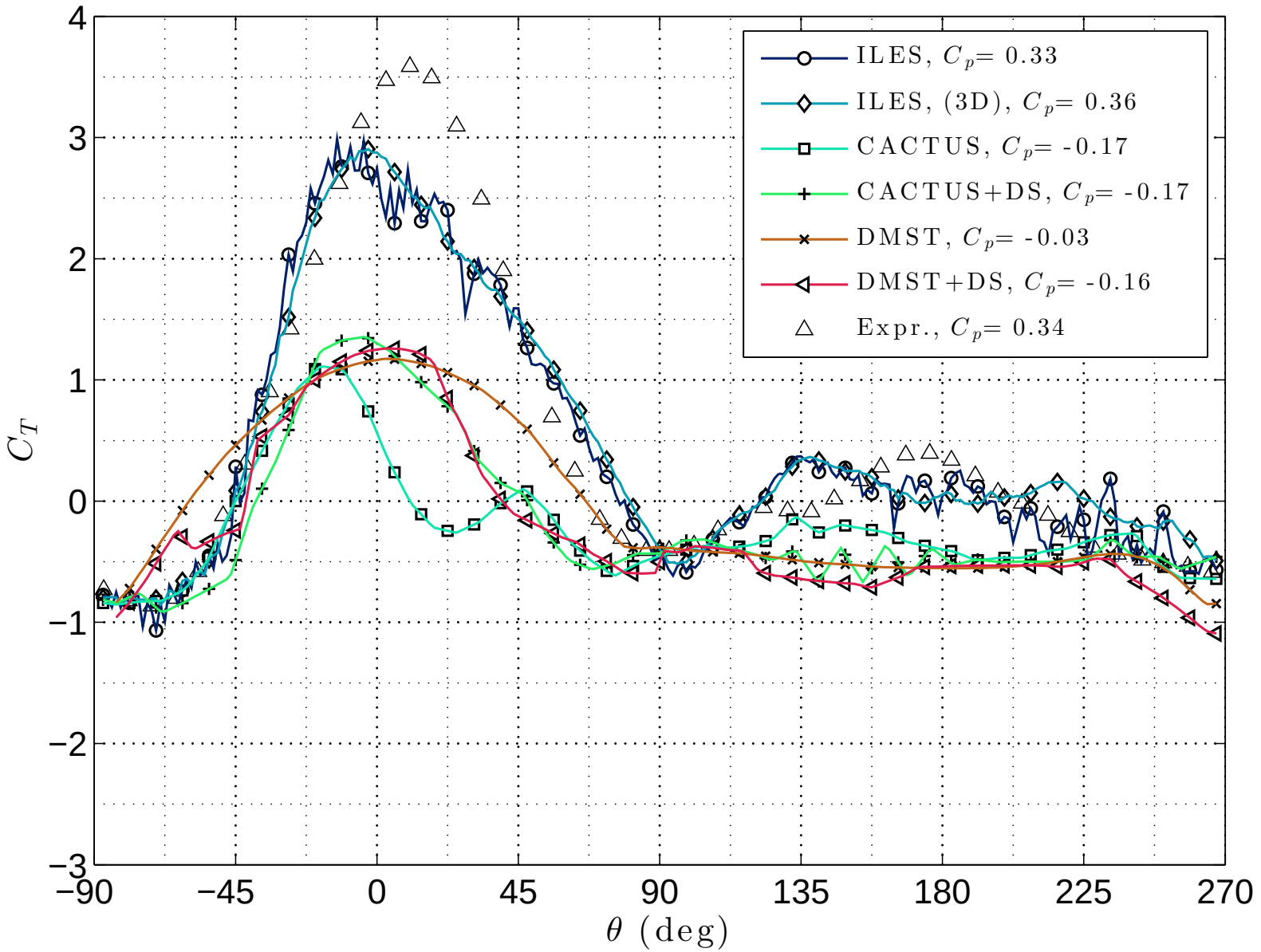
Results of SNL VAWT Simulations

ILES vs Analytical Models vs Experiments, λ=2.5



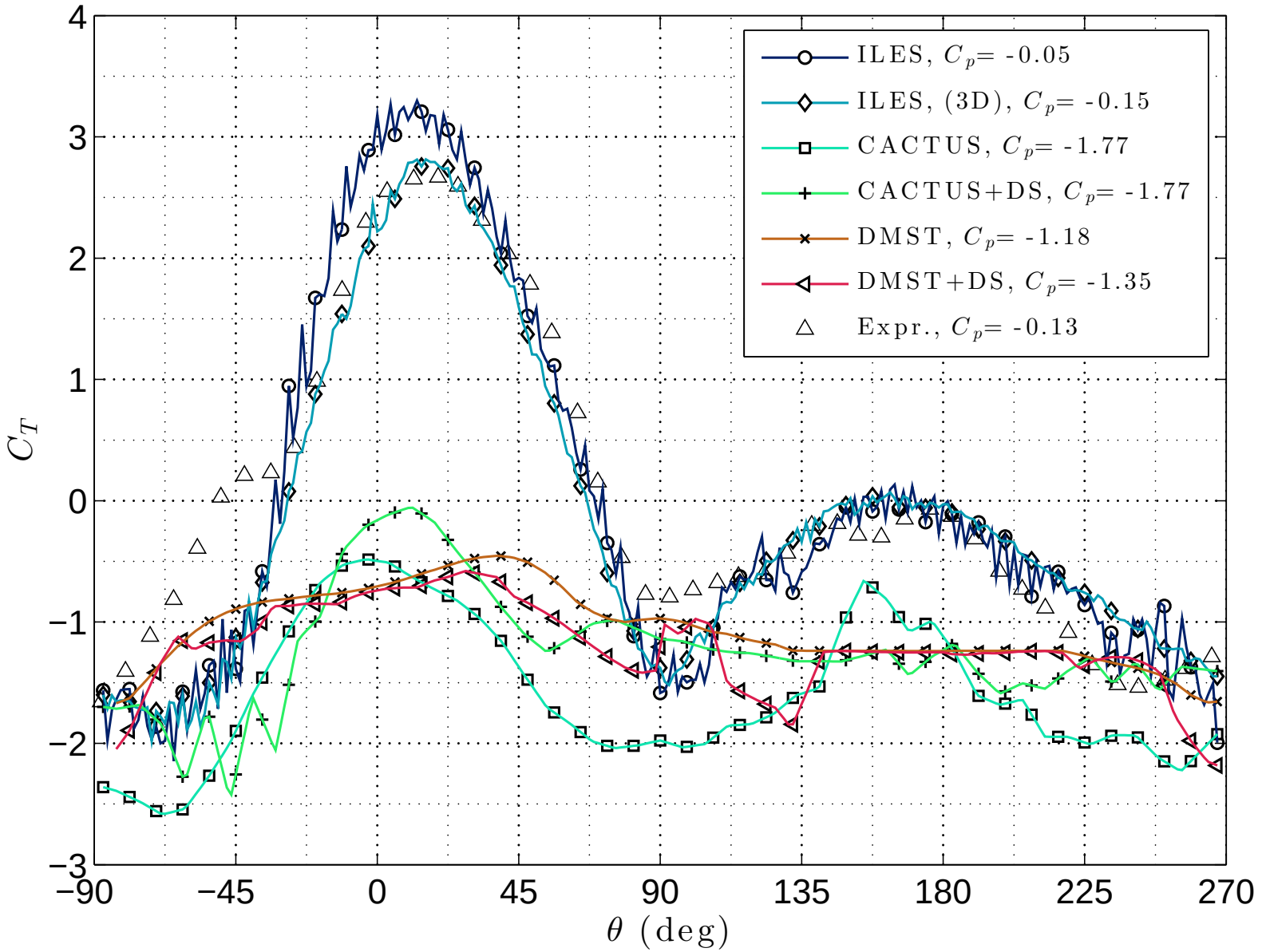
Results of SNL VAWT Simulations

ILES vs Analytical Models vs Experiments, λ=5.0



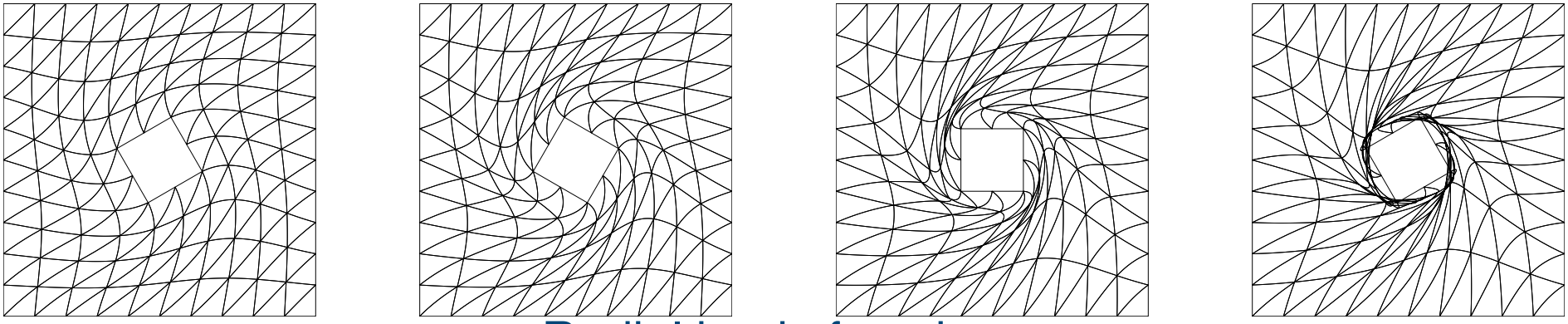
Results of SNL VAWT Simulations

ILES vs Analytical Models vs Experiments,

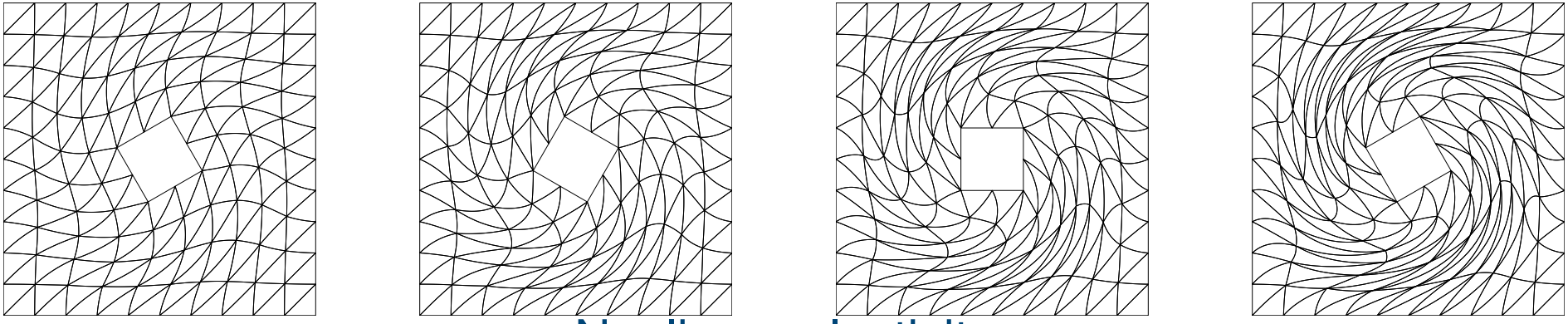


Domains with Large Deformations

- For large deformations, it is in general not possible to deform the meshes smoothly – *remeshing required*
- For efficient numerical schemes, use *local* mesh operations



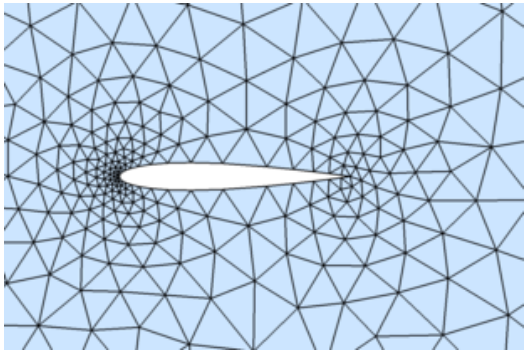
Radial basis functions

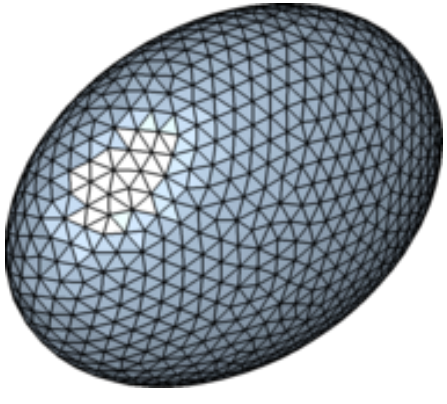


Nonlinear elasticity

The DistMesh Mesh Generator

- High quality meshes obtained using the *DistMesh* algorithm
  [Persson, Ph.D. thesis, 2005]
  1. Start with *any* topologically correct initial mesh
  2. Move nodes to find force equilibrium in edges
     - Project boundary nodes using *implicit geometry* $\phi(\boldsymbol{x})$
     - Update element connectivities with Delaunay
- Excellent properties:
  - Very simple (1 page of MATLAB)
  - Implicit geometries $\rightarrow$ No CAD required
  - Very high element qualities
  - Moving meshes/deforming domains
- Widely used:
  - Numerous books and courses
  - Rewritten in C, C++, C#, Fortran 77/90, Python, Mathematica, Octave





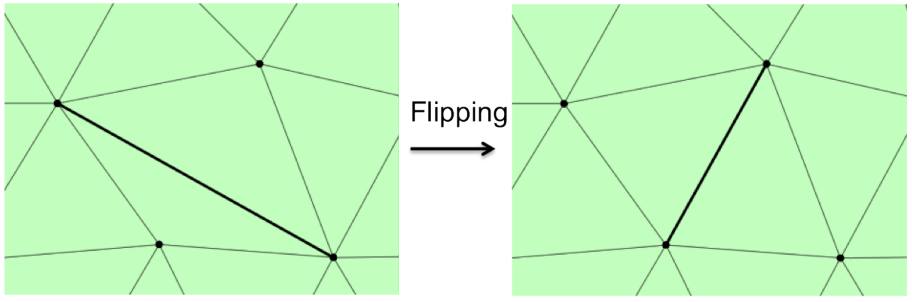
Mesh Motion and Edge Flipping

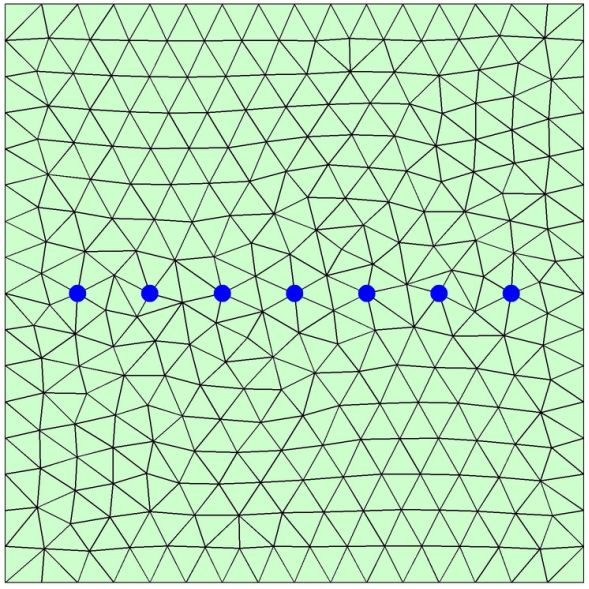
- Move the boundary nodes according to the prescribed geometry motion.
- Iteratively update the positions of interior nodes driven by spring-based force.

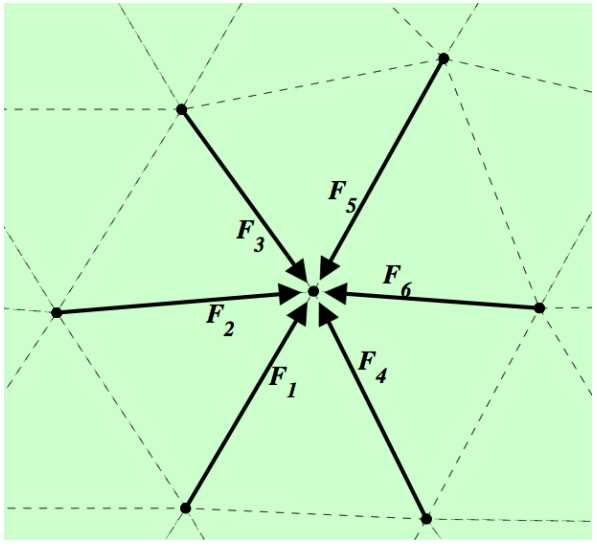
$$\boldsymbol{p}^{(n+1)} = \boldsymbol{p}^{(n)} + \delta \sum_i \boldsymbol{F}_i$$

$$|\boldsymbol{F}_i(l)| = \begin{cases} k(l - l_0) & \text{if } l \geq l_0, \\ 0 & \text{if } l < l_0, \end{cases}$$

- Edge Flipping. Each element can be flipped once during each time step

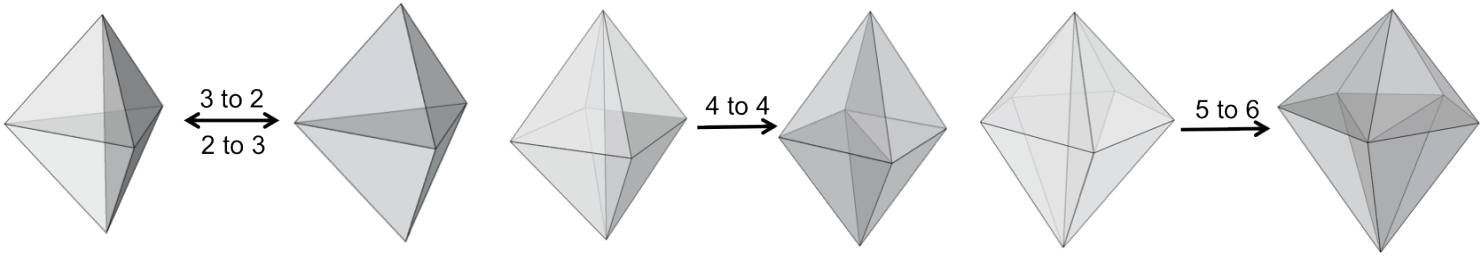




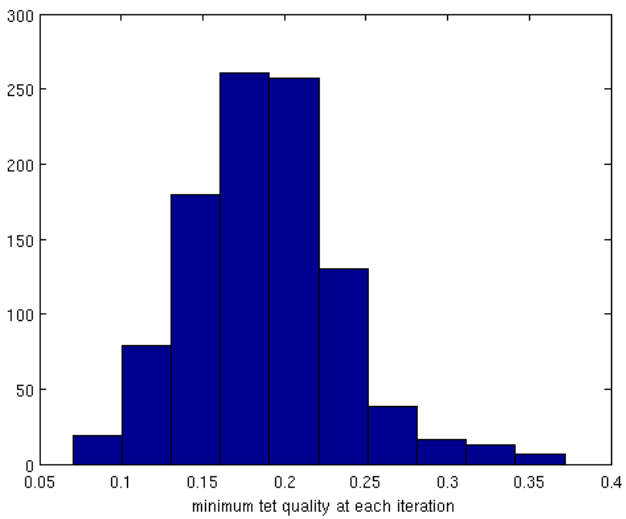


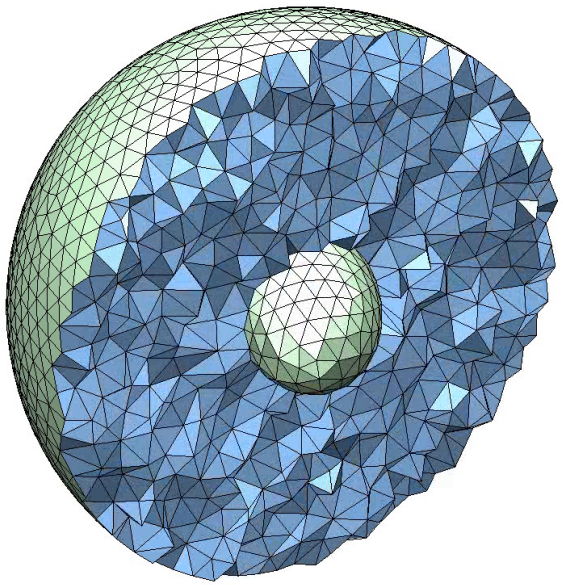
Element flipping in 3-D

- Local flipping in the 3D case:



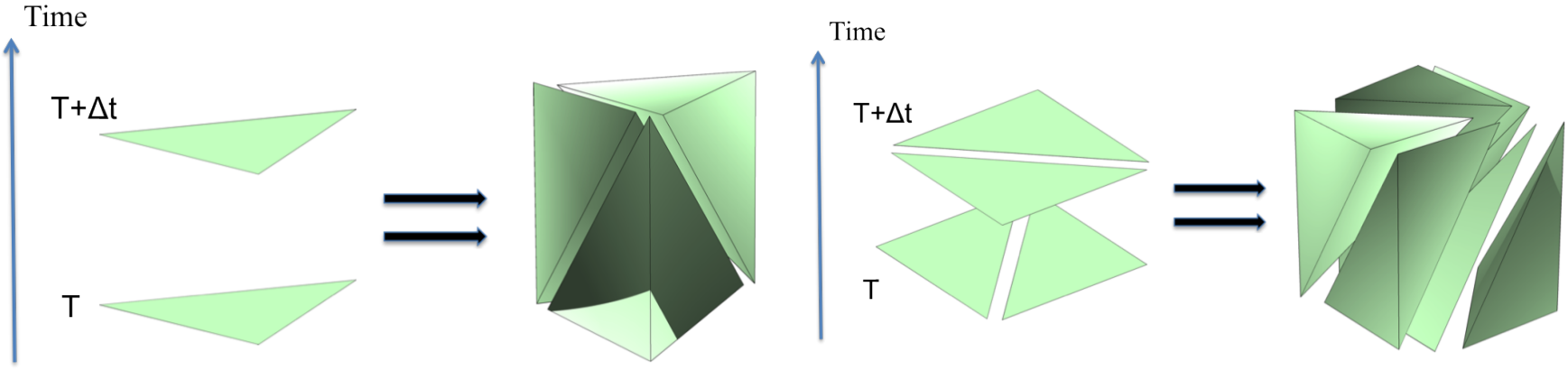
- The local flipping operations restrict the topology changes within a small number of elements





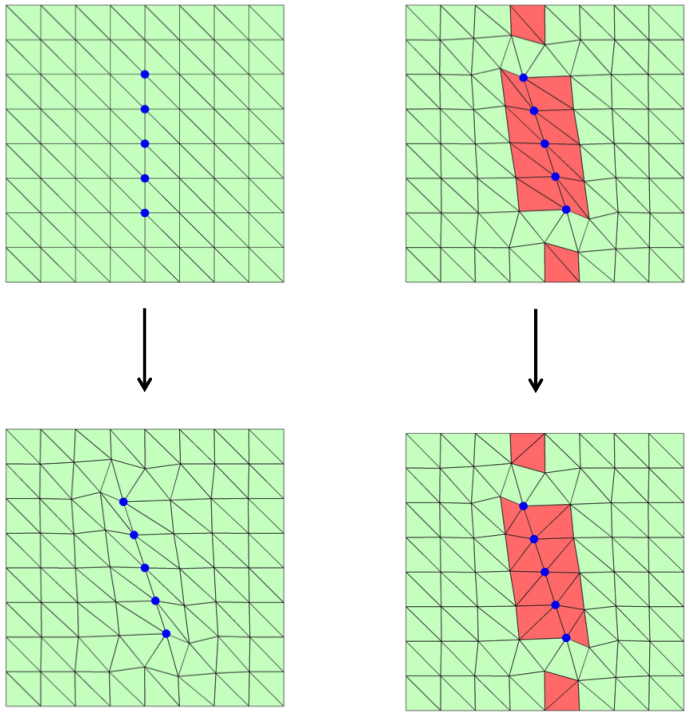
Time-stepping with topology changes

- Various approaches for high-order time-stepping with element flips
- Fully unstructured space-time DG method: [Wang/Persson 2013]
  - Fully consistent discretization in both space and time
  - Allows for arbitrary mesh deformations and topology changes
  - Local mesh operations significantly simplify the process of space-time slab mesh generation



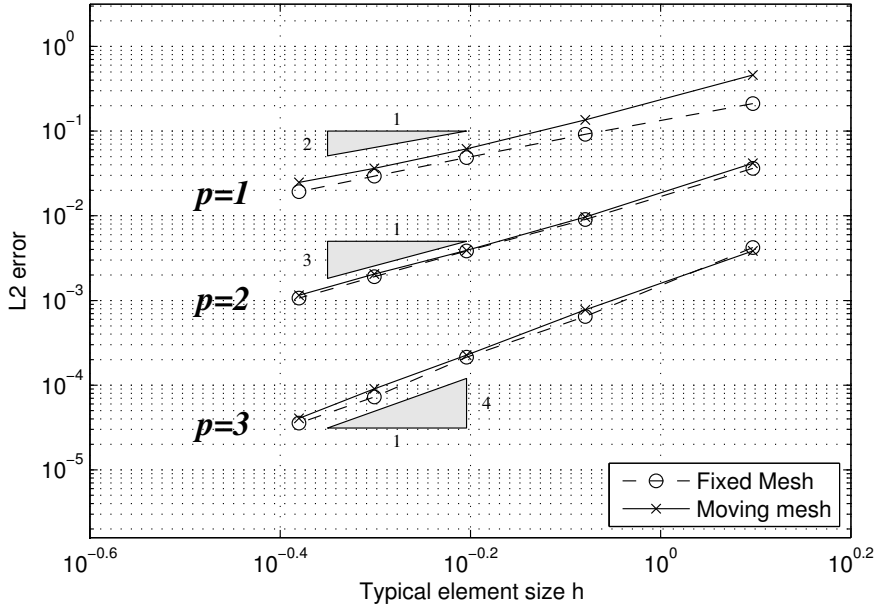
Time-stepping with topology changes

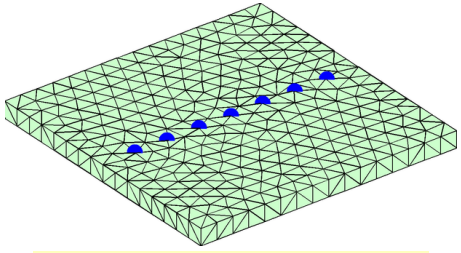
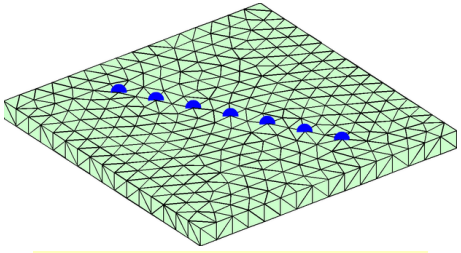
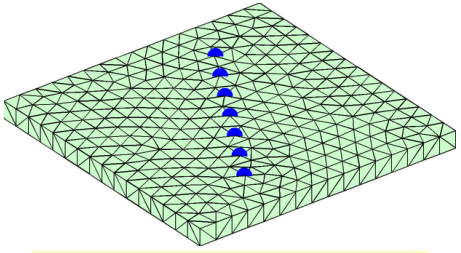
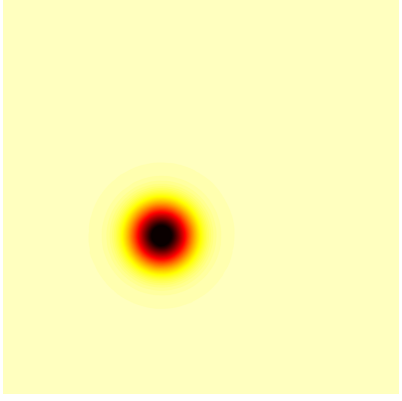
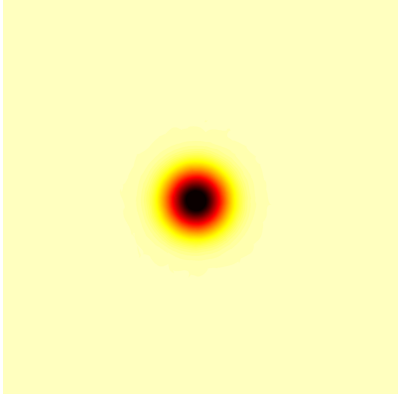
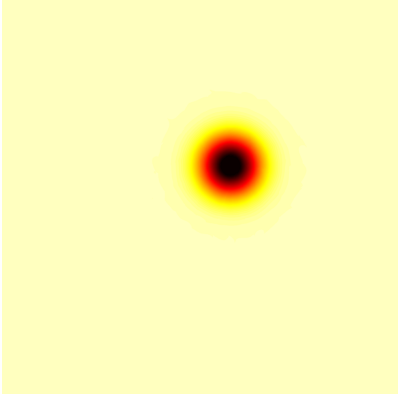
- Combination of the ALE formulation and local L^2 projections
  - For general re-meshing, this typically introduces significant errors and the implementation is complicated for general 3-D meshes
  - Instead, move nodes using the ALE formulation and apply local L^2 projections within each pair of red cells:



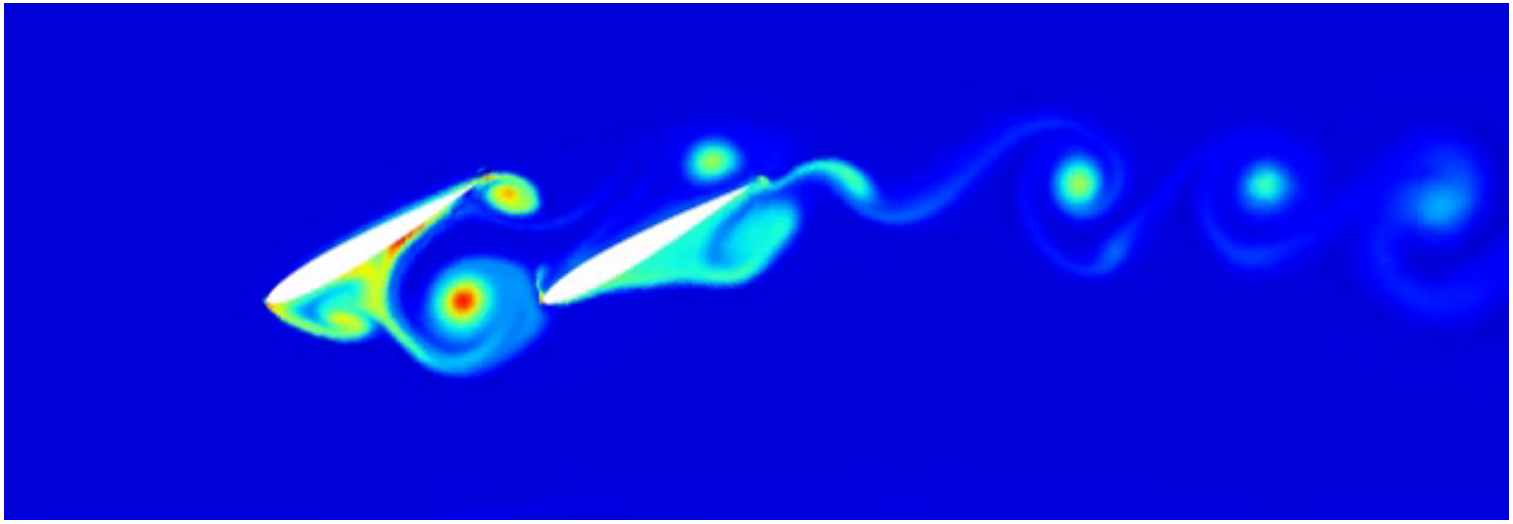
Example: Euler Vortex, Convergence

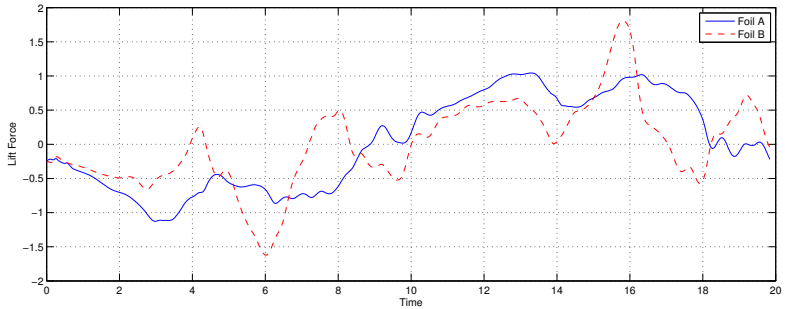
- Propagate an Euler Vortex on a fixed domain but moving mesh
- Optimal order of convergence $O(h^{p+1})$ for fixed and moving mesh.

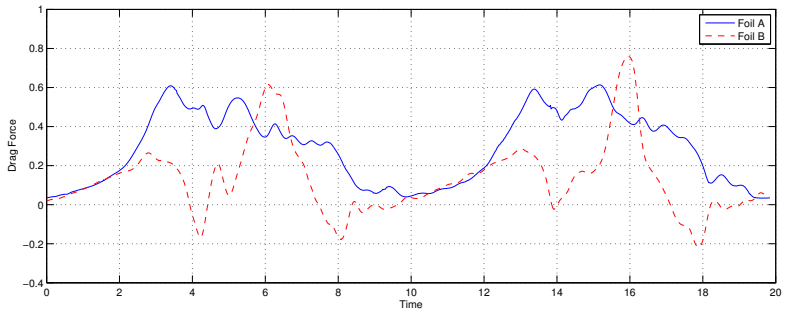


Example: Tandem Foils







Current work: Double VAWT Simulation

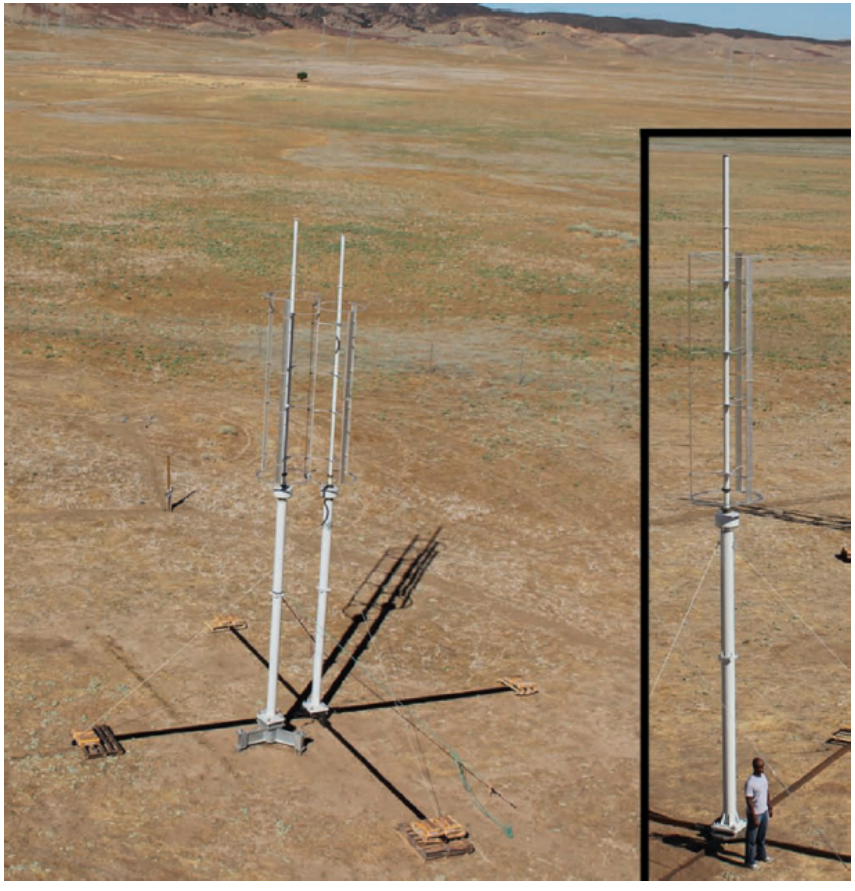


Figure: Counter-rotating VAWT experimental test site in the Antelope Valley, LA County, CA. (From Dabiri, 2011).

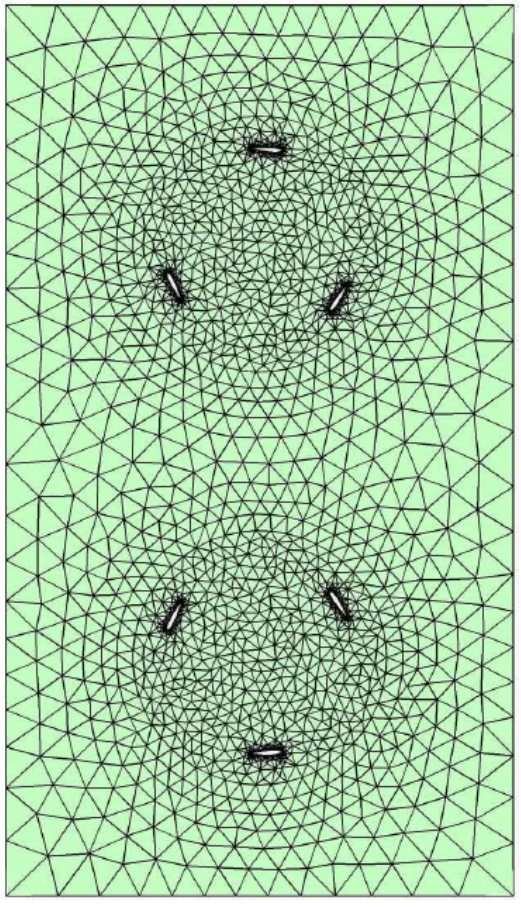


Figure: Computational domain for double VAWT simulation

Conclusion

- High-order LES with moving boundaries
- Moving mesh technique based on local mesh operations
- Applications in DNS/LES/DDES flow problems, flapping flight, wind turbine simulations, fluid-structure interaction, aeroacoustics