Spectral Difference Method for Sliding-Mesh Simulations, and Stratified Solar Convection

Chunlei Liang

joint work with Bin Zhang and Daniel Wang
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The George Washington University
Washington, DC
also with Mark Miesch of NCAR for the work of solar convection

AJ80th @Stanford
Nov. 21, 2014

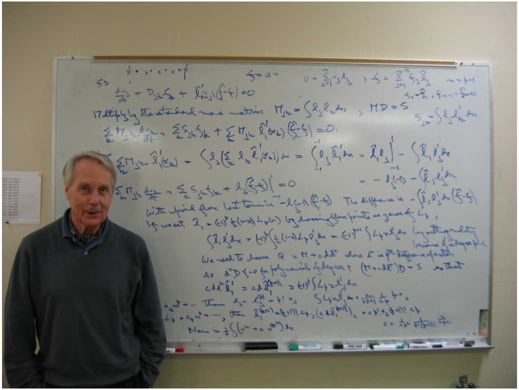
My academic career

1. postdoc @ Glasgow Scotland
2. hired by Antony in November 2007 @ Stanford
3. Since September 2010 Assistant Professor @ George Washington University

Section 1

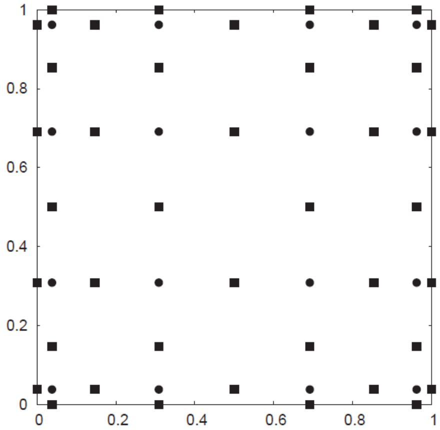
Thanks to Professor Jameson's Work on High-order Methods

Proof of the stability of the spectral difference method



Proof was done in early 2009 published in JSC 2010

Spectral Difference Method



1. Flux points: Legendre polynomial roots and two end points
2. Solution points: Chebyshev Gauss points
3. Tensor product

Other topics Prof. Jameson has worked on

- New Flux Reconstruction Method
- High-order Methods for Large Eddy Simulation
- High-order Methods for Complex Geometries
- High-order Methods for Moving Boundary Problems
- GPUs

Special Issue 2014

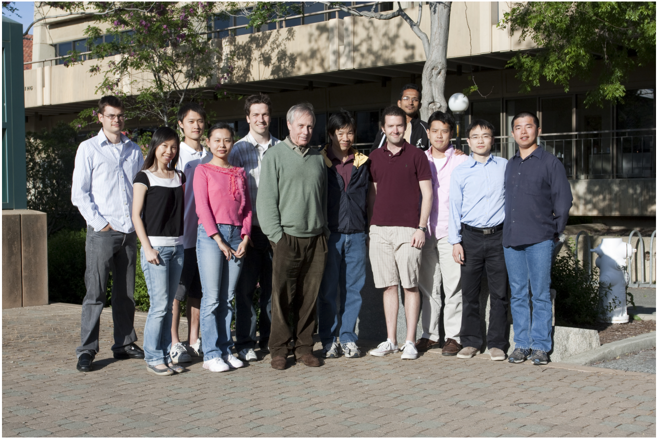
- Special Issue of High-order Methods for Computational Fluid Dynamics

  in honor of Professor Jameson's 80th Birthday

  published by Computers & Fluids in 2014

  edited by Chunlei Liang, Krzysztof Fidkowski, Per-Olof Persson and Peter Vincent

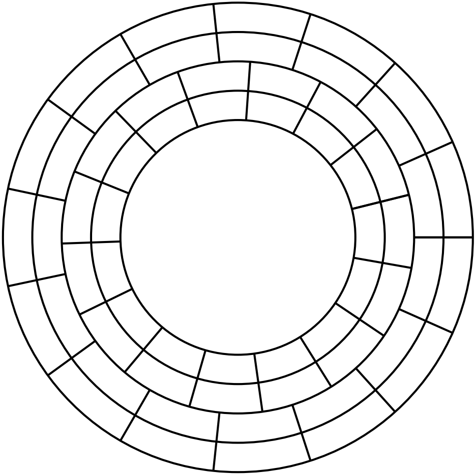
ACL 2009



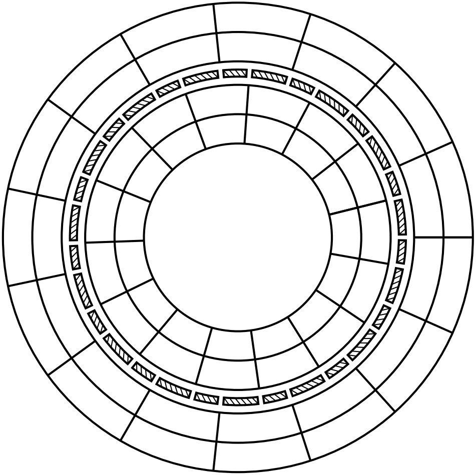
Section 2

Spectral Difference Method for Sliding-Mesh Simulations

Sliding-mesh interface



Dynamic construction of curved mortars



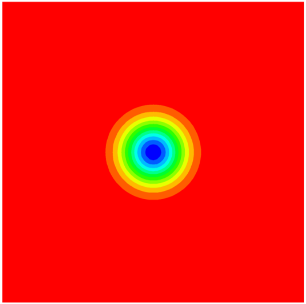
Euler vortex: problem setup

$$u = U_\infty \left\{ \cos\theta - \frac{\epsilon y_r}{r_c} \exp\left(\frac{1 - x_r^2 - y_r^2}{2r_c^2} \right) \right\}$$

$$v = U_\infty \left\{ \sin\theta + \frac{\epsilon x_r}{r_c} \exp\left(\frac{1 - x_r^2 - y_r^2}{2r_c^2} \right) \right\}$$

$$\rho = \rho_\infty \left\{ 1 - \frac{(\gamma-1)(\epsilon M_\infty)^2}{2} \exp\left(\frac{1 - x_r^2 - y_r^2}{r_c^2} \right) \right\}^{\frac{1}{\gamma-1}}$$

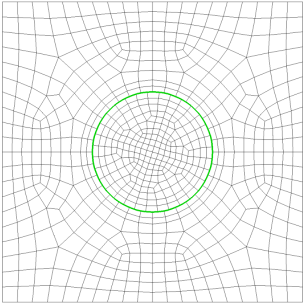
$$p = p_\infty \left\{ 1 - \frac{(\gamma-1)(\epsilon M_\infty)^2}{2} \exp\left(\frac{1 - x_r^2 - y_r^2}{r_c^2} \right) \right\}^{\frac{\gamma}{\gamma-1}}$$

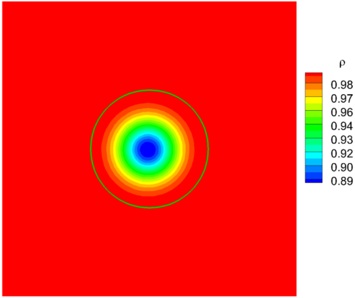


In this test we set $(U_\infty, \rho_\infty, p_\infty) = (1,1,1)$, $M_\infty = 0.3$, $\theta = \arctan(1/2)$, $\epsilon = 1$, $r_c = 1$.

Domain size is $0 \leq x, y \leq 10$, initial location of vortex is $(x_0, y_0) = (5,5)$. Periodic boundary condition is used.

Euler vortex: animations





Results from 4*th* order scheme:

Euler vortex: order of accuracy

cells	L1 error	order	L2 error	order
180	3.16E-4	-	8.13E-4	-
700	4.58E-5	2.85	1.13E-4	2.90
2731	7.00E-6	2.80	1.73E-5	2.83

Table: Error and order of accuracy of the 3^{rd} order scheme

cells	L1 error	order	L2 error	order
180	5.43E-5	-	1.26E-4	-
700	3.27E-6	4.14	8.02E-6	4.06
2731	2.26E-7	4.03	5.50E-7	4.00

Table: Error and order of accuracy of the 4^{th} order scheme

Taylor-Couette flow: problem setup

The exact solution of the velocity is

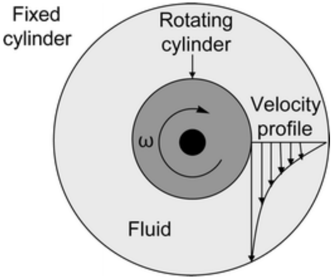
$$v_\theta = \omega r_1 \frac{r_2/r - r/r_2}{r_2/r_1 - r_1/r_2} \quad (1)$$

In this test we choose $\omega = 1$, $r_1 = 1$, $r_2 = 2$.

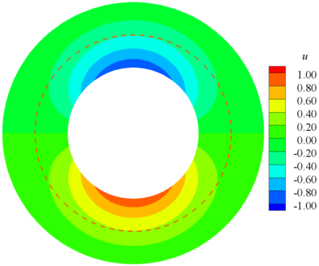
Mach number on the inner wall is $Ma = 0.1$.

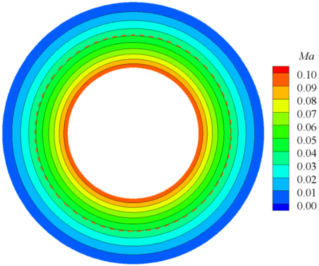
Reynolds number based on inner cylinder speed and radius is $Re = 10$.

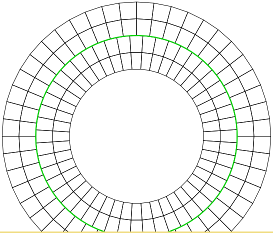
Isothermal wall boundary condition is used for both cylinders.



Taylor-Couette flow: meshing and flow







Taylor-Couette flow: Order of accuracy

cells	L1 error	order	L2 error	order
192	5.90E-5	-	8.71E-5	-
768	6.95E-6	3.09	9.82E-6	3.15
3072	8.43E-7	3.07	1.09E-6	3.16

Table: Error and order of accuracy of the 3^{rd} order scheme

cells	L1 error	order	L2 error	order
192	9.72E-6	-	1.52E-5	-
768	6.16E-7	3.98	1.01E-6	3.91
3072	4.22E-8	3.92	6.60E-8	3.92

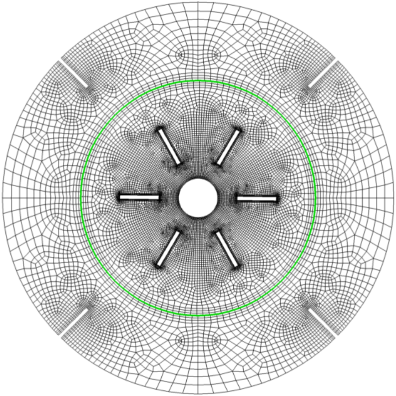
Table: Error and order of accuracy of the 4^{th} order scheme

What's good with this method?

1. Simple
2. High-order accurate
3. Efficient
4. Easy to parallelize

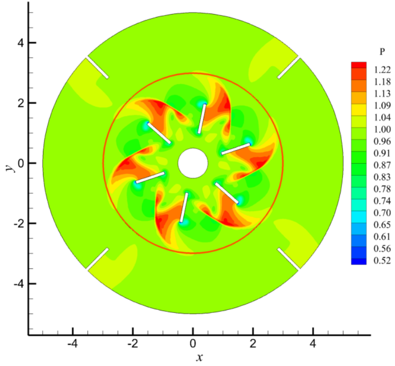
2D stirred vessel: setup

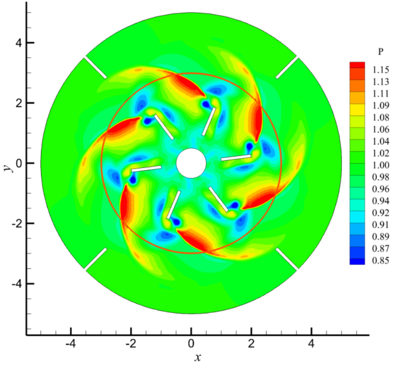
$\omega = 1$
$Re = \omega D^2/\nu = 100$, D is inner cylinder diameter
$Ma = 0.1$ on inner wall
adiabatic wall bc on blades
isothermal wall elsewhere
mesh has 14990 cells



2D stirred vessel: pressure contours

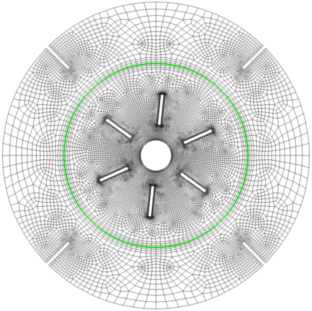
initial period (3*rd* order scheme):

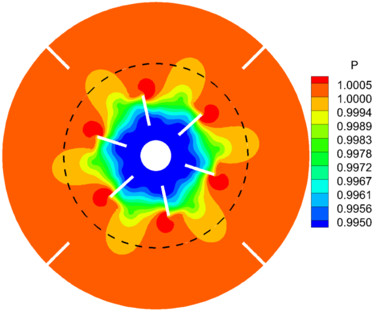




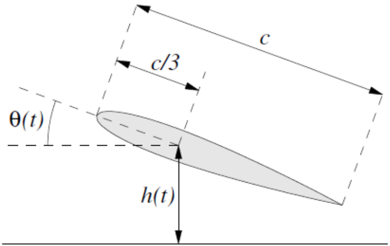
2D stirred vessel: animations

converged solution (3*rd* order scheme):





Heaving and pitching airfoil: setup



NACA 0012 airfoil

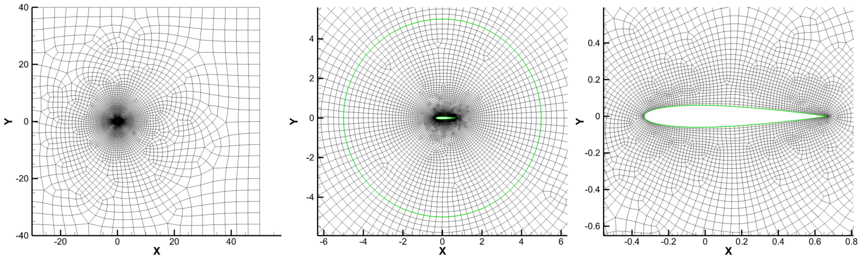
$h = A\cos(2\pi\omega t)$, where $A = 0.25$, $\omega = 0.4$

$\theta = \alpha\sin(2\pi\omega t)$, where $\alpha = 60°$

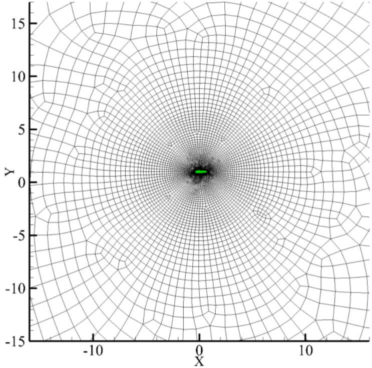
$Re = U_\infty c/\nu = 1000$

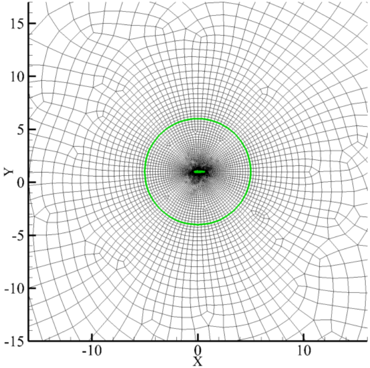
Heaving and pitching airfoil: Meshing

Mesh with 9315 cells:



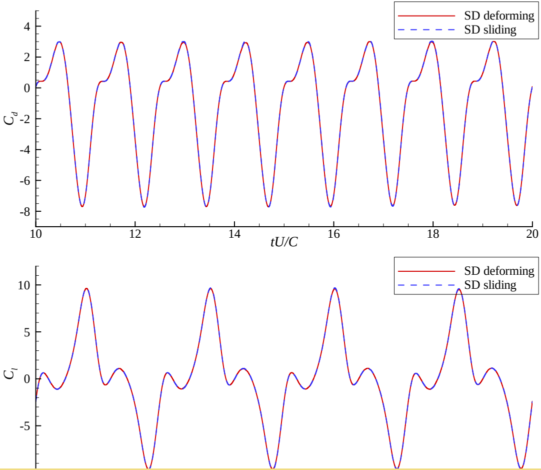
Heaving and pitching: animations



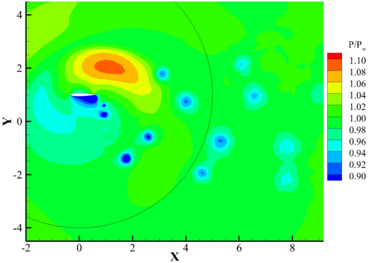


Heaving and pitching airfoil: force coefficients

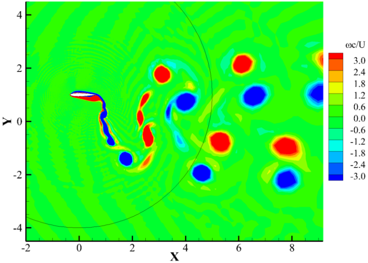
Results from 3*rd* order scheme:



Heaving and pitching airfoil: animations



(a)Pressure



(b) Vorticity

Section 3

Spectral Difference Method for Solar Convection

Differential Rotation (Helioseismology)

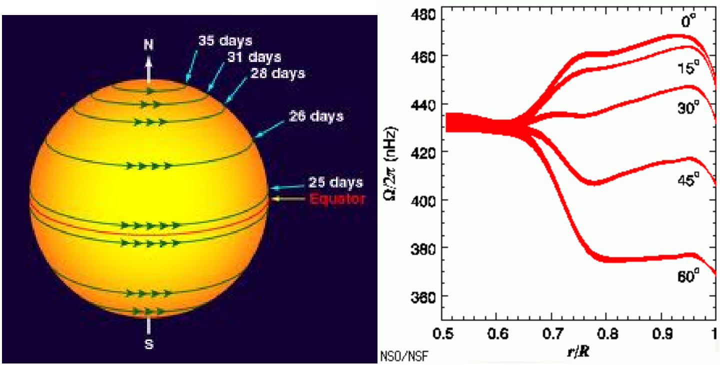


Figure: Differential rotation

Solar Interior

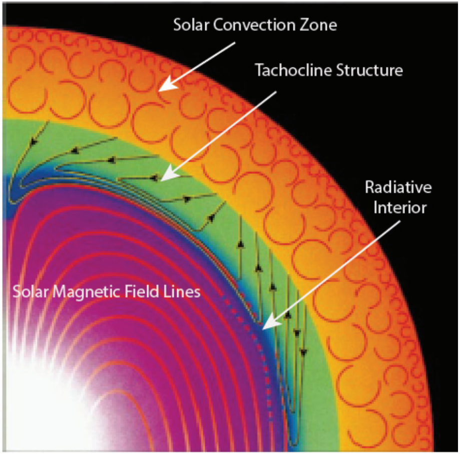


Figure: Gough and McIntyre, Nature, 1998

Variable-resolution meshing technique

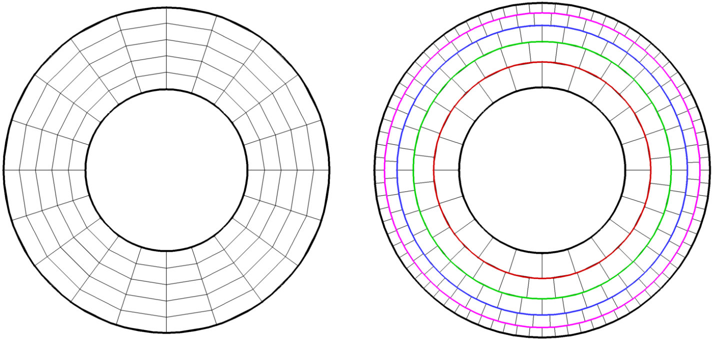


Figure: Conforming mesh (a) and Nonconforming mesh with variable resolution (b)

Fully compressible flow Model in a rotating reference frame

Rotates with reference to z axis at a rate of $\boldsymbol{\Omega_o}$.
The hydrodynamic equations are:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{u}) \tag{2}$$

$$\frac{\partial (\rho \boldsymbol{u})}{\partial t} = -\nabla \cdot \rho \boldsymbol{u}\boldsymbol{u} - \nabla p + \nabla \cdot \boldsymbol{\tau} + \rho \boldsymbol{g} - 2\rho \boldsymbol{\Omega}_0 \times \boldsymbol{u} \tag{3}$$

$$\frac{\partial (E)}{\partial t} = -\nabla \cdot ((E + p)\boldsymbol{u}) + \nabla \cdot (\boldsymbol{\tau} \cdot \boldsymbol{u} - \boldsymbol{f}) + \rho \boldsymbol{u} \cdot \boldsymbol{g} \tag{4}$$

where E is the total energy per unit volume, $\boldsymbol{\tau}$ is viscous stress tensor, $\boldsymbol{f}$ is entropy/radiative/conductive flux and $\boldsymbol{g}$ is gravitational acceleration.

What make the implementation hard?

1. Hydrostatic balance:

$$\frac{dp_e}{dr} = -\rho_e \boldsymbol{g} \tag{5}$$

2. Thermal equilibrium:

$$\frac{d}{dr}\left(r^2 \rho_e T_e \kappa \frac{ds_e}{dr}\right) = 0 \tag{6}$$

3. Radiation plays an important role for energy transport
4. Angular momentum conservation. (Implicit constraints during time stepping).

Verify fully compressible codes

In need of

creating new benchmark cases for stratified convection in spherical shells!

1. reduce Reynolds number
2. reduce Rayleigh number
3. spin faster
4. increase luminosity of the star
5. neglect the near surface shear layer

Unstructured Grid

Total cells: 294, 912; Total DOFs: 18,874,368.

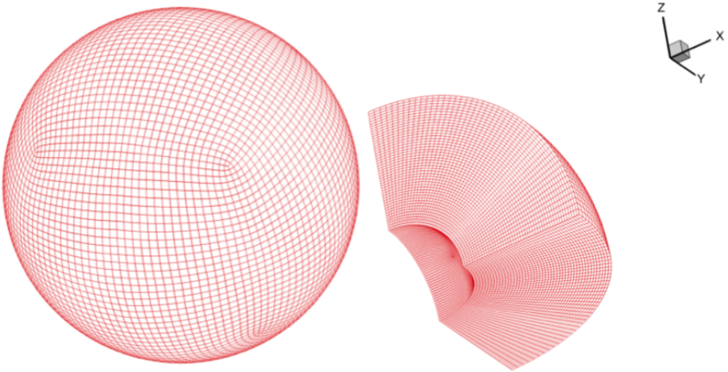
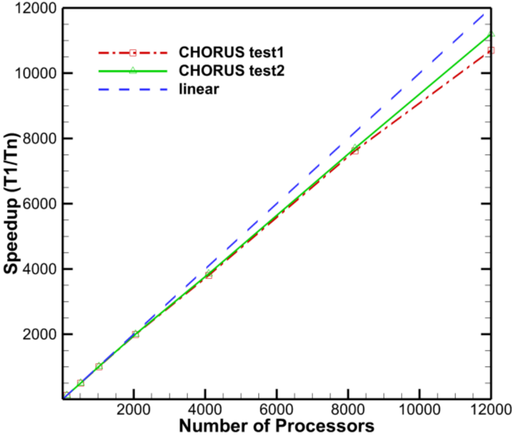


Figure: Unstructured Hexahedral Grid

Scalability of CHORUS



Initial equilibrium conditions

- Hydrostatic balance:

$$\frac{dp_e}{dr} = -\rho_e \boldsymbol{g} \tag{7}$$

- Thermal equilibrium:

$$\frac{d}{dr}(r^2 \rho_e T_e \kappa \frac{ds_e}{dr}) = 0 \tag{8}$$

- Ideal gas law:

$$p = \rho RT \tag{9}$$

Boundary Conditions

- Impenetrable

$$u_r = 0 \tag{10}$$

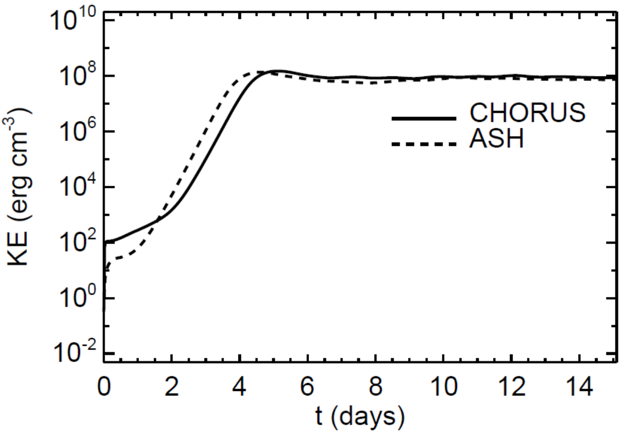
- Stress-free

$$\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) = \frac{\partial}{\partial r}\left(\frac{u_\phi}{r}\right) = 0 \tag{11}$$

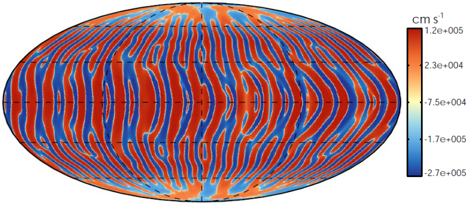
  Angular momentum conservation is re-enforced during every rotational cycle.

- A constant heat flux $\mathbf{f}_b$ at the bottom
- The temperature at the top is fixed.

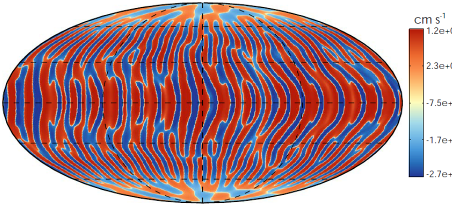
Time history of the total kinetic energy: Solar model



Mollweide view of radial velocity for the Sun simulation



(a) Compressible code



(b) NCAR ASH code

Mollweide view of radial velocity at $0.95 R_{\odot}$

$(loadingsolarmovie)$

This is the first time that an unstructured grid code successfully predicts the differential rotation of the sun!

Ongoing work

1. 3D simulation of a marine propeller
2. Oblate stars

Submitted manuscripts

1. B. Zhang and C. Liang, A simple, high-order accurate sliding interface approach to spectral difference method for predicting compressible viscous flow on coupled rotating and stationary domains. Submitted to Journal of Computational Physics, September 2014.
2. J. Wang, C. Liang, M. S. Miesch, A Compressible High-Order Unstructured Spectral Difference Code for Stratified Convection in Rotating Spherical Shells, submitted to Journal of Computational Physics, July 2014.

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PhD student Daniel Junfeng Wang has been supported by a Newkirk Graduate Fellowship from:

Happy Birthday to Antony!

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