# The continuous adjoint strikes back

AJ80TH

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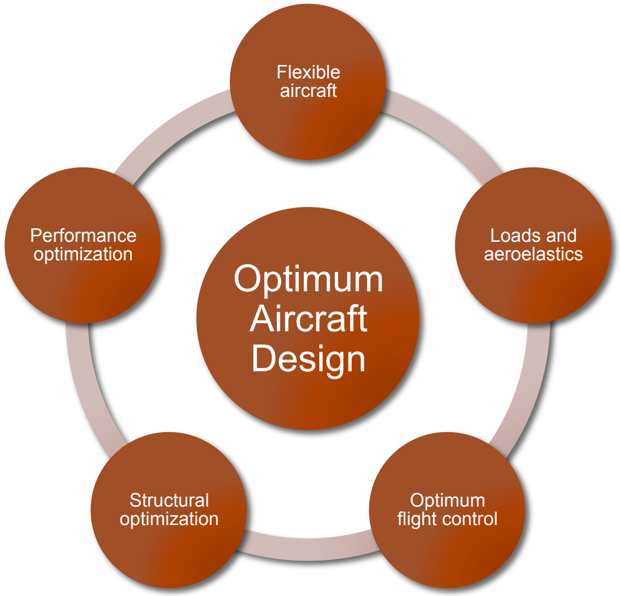


Flexible aircraft
Loads and aeroelastics
Optimum flight control
Structural optimization
Performance optimization
Optimum Aircraft Design

# The continuous adjoint strikes back or… the revenge of the continuous adjoint (a personal vision)

- The **aerodynamic optimal shape methodology using control theory was discovered and developed by Prof. Jameson** (~100 articles). From theory to complex applications.
- Prof. Jameson mainly worked on the continuous and the discrete (by hand) adjoint. And, in both cases, **he demonstrated that the method was mature to deal with complex wing body configurations**.

Then… something happened…

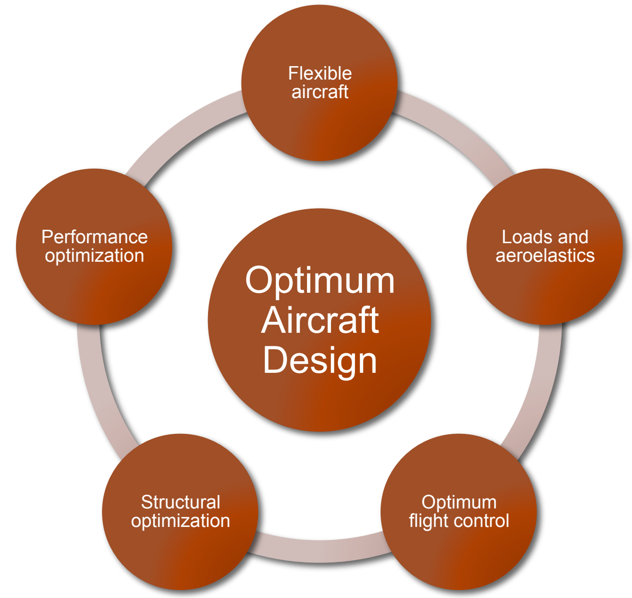
- **The CFD community started looking at the Automatic Differentiation** based techniques and we entered in a stall situation where the new “more accurate” techniques were not able to deal with complex problems, putting a brake on the industrial application.

Now... just few **industrial CFD adjoint solvers** are able to reproduce optimizations done by Prof. Jameson 10-15 years ago… It is time to rediscover the continuous adjoint methodology and exploit its advantages.

# Why aerodynamic shape optimization is so important?

We what to tackle some of the key aeronautical challenges.

- Quality and Affordability.
- Environment (low fuel burn, low emissions levels, low noise levels)
- Safety.
- Efficiency of the Air Transport System.
- Security.

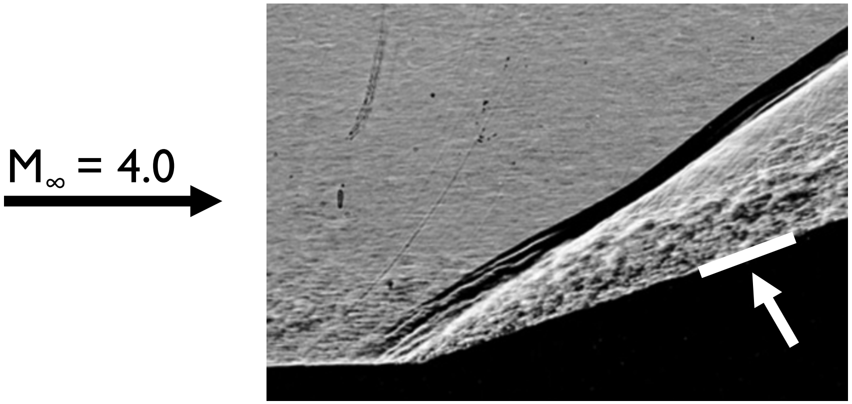



- Full multi-disciplinary design optimization (aerodynamic drag minimization, weight saving through loads management, ...).

- Speeding up the design process is the enabler for more optimized design of complete aircraft ➔ Radically change the design process and the role of the engineer.

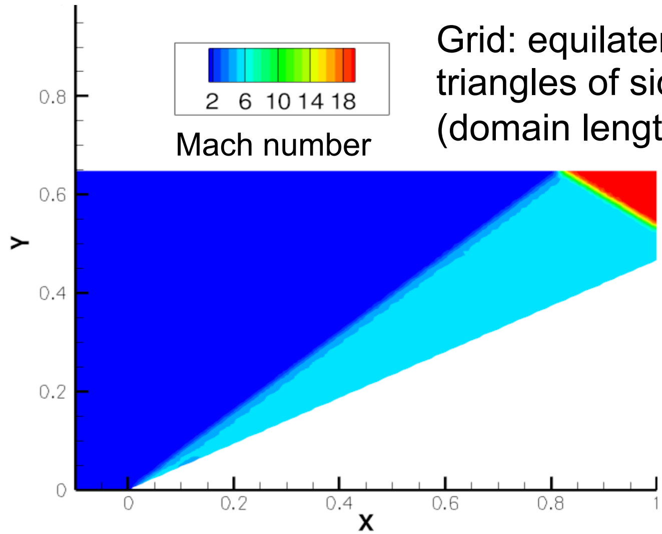
# Reality or myth about the adjoint methodology

- The continuous adjoint gradients are inaccurate. ??
- The Automatic Differentiation uses a large amount of memory and resources. ??
- Automatic Differentiation is the solution in complex physics. ??
- Surface formulation only makes sense for the continuous adjoint. ??
- Automatic Differentiation can deal with cost functions of arbitrary complexity. ??
- Connection and arrangement of the differentiated routines to compute the discrete adjoint is the best solution. ??

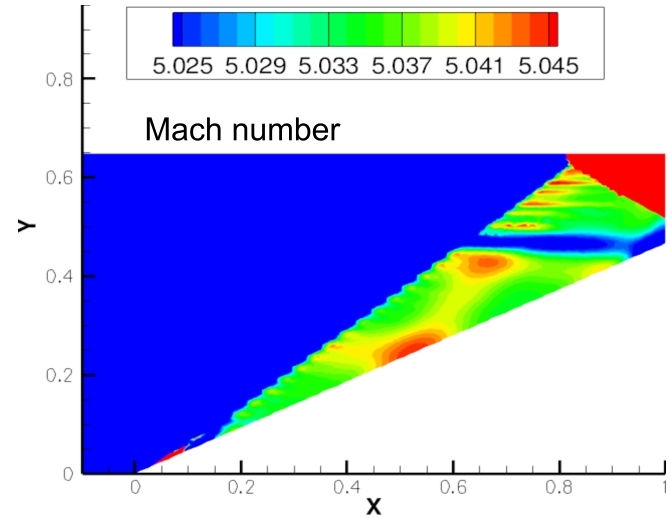


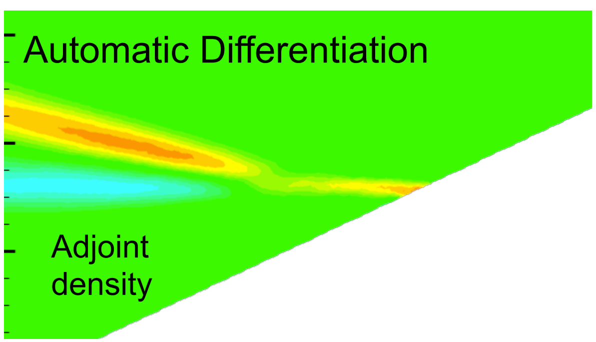

Suppose we want to compute the derivative of the pressure in this patch as a function of the free stream Mach number

# Reality or myth about the adjoint methodology



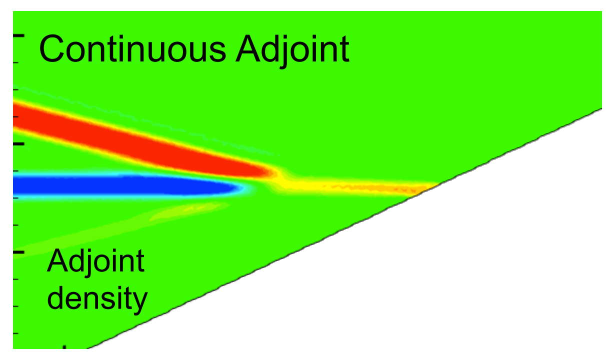

Grid: equilateral triangles of side = 0.01 (domain length ~ 5 units)




0% diff. with respect to discrete sensitivity

12% diff. with respect to analytical sensitivity




10 % diff. with respect to discrete sensitivity

0.1 % diff. with respect to analytical sensitivity

# In this talk

- The objective of this talk is to present an overview of **challenges and opportunities for the continuous adjoint methodology.**
- **Highlighting those areas in which the continuous adjoint makes a difference** from the industrial point of view:
  - Fully exploit the surface formulation, understand the limitations and advantages.
  - Complex systems, more complex equations, more complex geometries, larger problems.
  - Efficient unsteady design.
  - Design with discontinuities.

INTRODUCTION

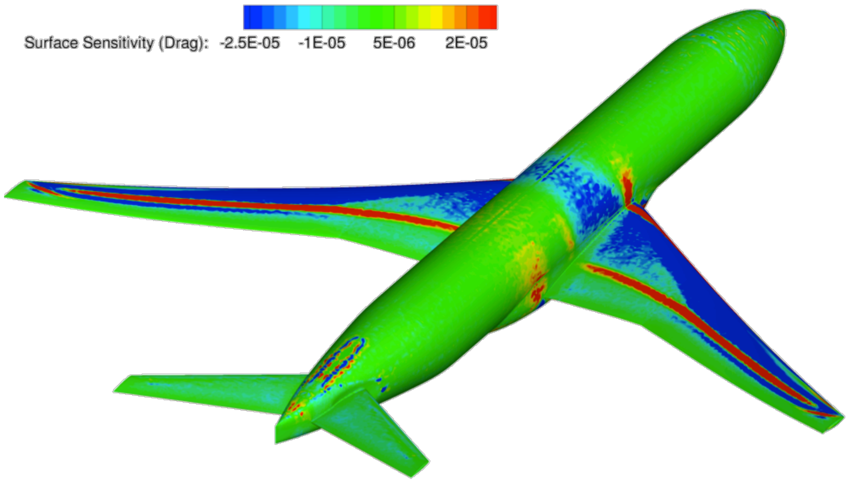
**SURFACE FORMULATION**

COMPLEX SYSTEMS

UNSTEADY DESIGN

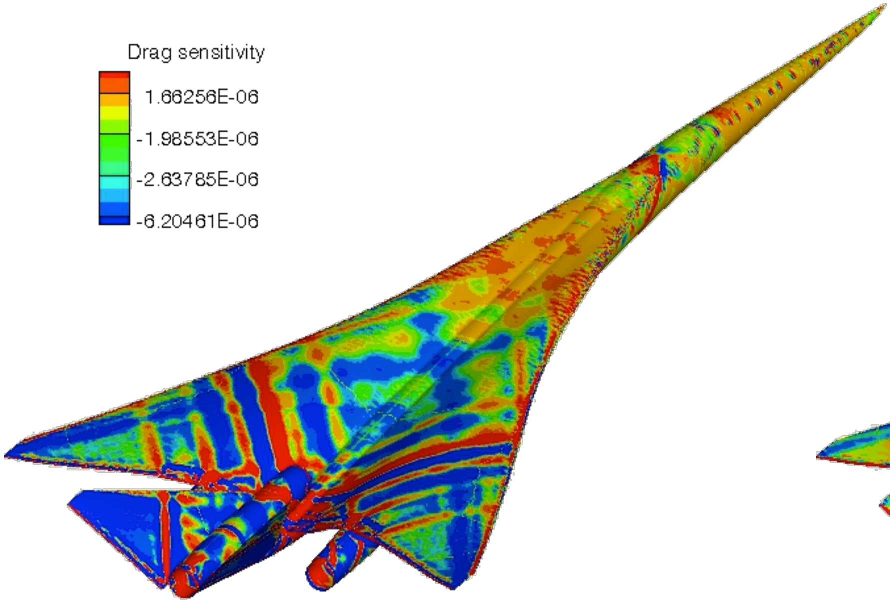
DESIGN WITH DISCONTINUITIES

CONCLUSIONS

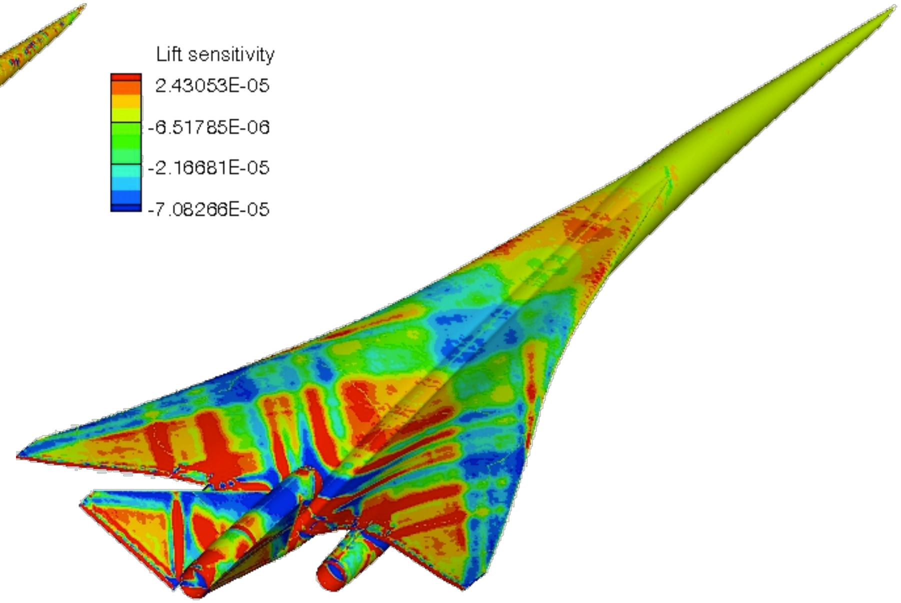


# What is the surface formulation?

The **Continuous Adjoint** provides surface sensitivities: a measure of the change in the objective function at each node due to small perturbations in the local normal direction.




Drag sensitivity (Mach 1.6, AoA 2.3deg)

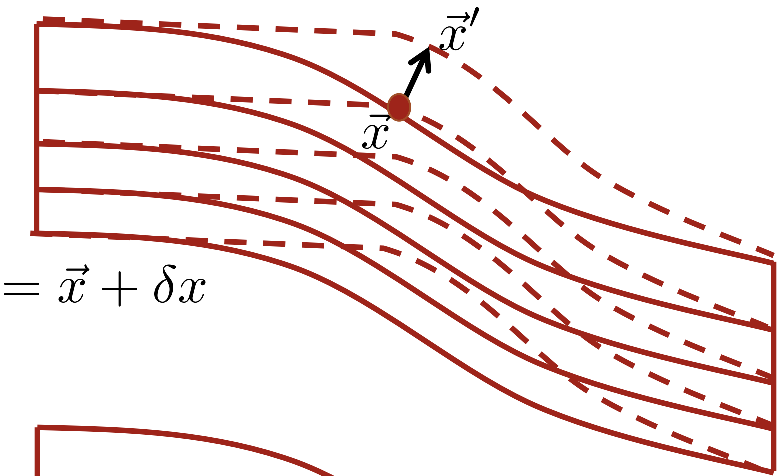



Lift sensitivity (Mach 1.6, AoA 2.3deg)

Designers can use this sensitivity information to determine appropriate parameterizations of the configuration prior to optimization.

# Reduced Gradient Formulation

If we change the boundary in a PDE problem. How to compute the variation of a quantity *U*?

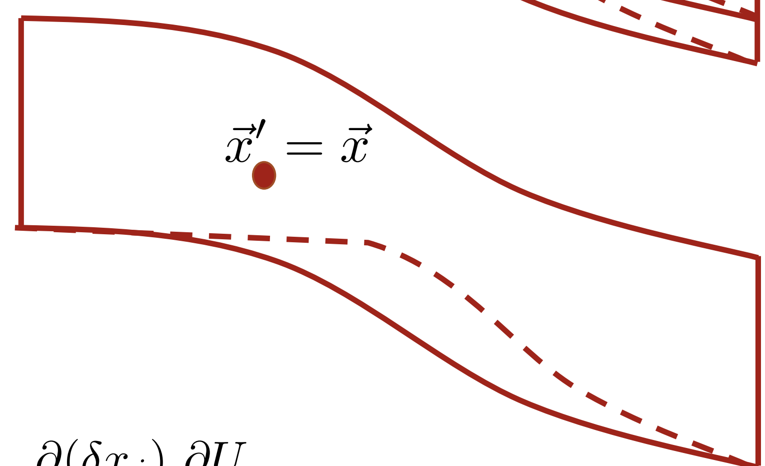



- The point will move with the boundary movement (implicit reference to an underlying computational mesh).

$$\delta U(\vec{x}) = U'(\vec{x}'(\vec{x})) - U(\vec{x}), \text{ where } \vec{x}' = \vec{x} + \delta x$$

- The point location is fixed even if we move the boundary (frozen Cartesian space).

$$\hat{\delta} U(\vec{x}) = U'(\vec{x}) - U(\vec{x})$$




They seem similar but the spatial derivative is different.

$$\frac{\partial(\hat{\delta} U)}{\partial x_i} = \hat{\delta}\left(\frac{\partial U}{\partial x_i}\right) \qquad \frac{\partial(\delta U)}{\partial x_i} = \delta\left(\frac{\partial U}{\partial x_i}\right) + \frac{\partial(\delta x_j)}{\partial x_i}\frac{\partial U}{\partial x_j}$$

# Reduced Gradient Formulation

When we introduce the variation of the governing equations using $\delta$

$$\delta I = \int_{\delta B} d_i n_i P \, dB + \int_B d_i n_i \delta P \, dB - \int_D \Psi^T \delta R \, dD$$

This is a mechanism to account for the grid movement

$$\delta I = \int_{\delta S} d_i n_i P \, dS + \int_S d_i n_i \delta P \, dS - \int_D \Psi^T \frac{\partial(\delta F_i)}{\partial x_i} \, dD + \boxed{\int_D \Psi^T \frac{\partial(\delta x_j)}{\partial x_i} \frac{\partial F_i}{\partial x_j} \, dD}$$

But… maybe we don't want that term… Reduced gradient formulation by Prof. Jameson

$$\frac{\partial(\delta x_j)}{\partial x_i} \frac{\partial F_i}{\partial x_j} = \frac{\partial}{\partial x_i} \left( \delta x_j \frac{\partial F_i}{\partial x_j} \right)$$

and

$$\int_\Omega \Psi^T \delta R \, dD = \int_B \Psi^T n_i \left( \delta - \delta \vec{x} \cdot \vec{\nabla} \right) F_i \, dB$$

Obtaining a pure boundary based formulation and the same result if you use $\hat{\delta}$

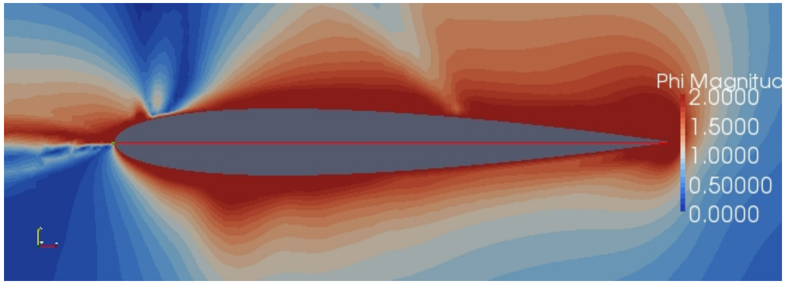
$$\delta I = \int_{\delta B} d_i n_i P \, dB + \int_B d_i n_i \left( \hat{\delta} P + \delta x_i \frac{\partial P}{\partial x_j} \right) \, dB - \int_\Omega \Psi^T \hat{\delta} R \, dD$$

# Surface sensitivity formulae

## Surface sensitivity Euler's case

$$\delta J(S) = \int_{S\setminus x_b} \left[\frac{\partial j}{\partial P}\partial_n P + \mathbf{t}\cdot\partial_{tg}\left(\frac{\partial j}{\partial \mathbf{n}_S}\right) - \kappa\left(j + \frac{\partial j}{\partial \mathbf{n}_S}\mathbf{n}_S\right)\right]\delta S\,\mathrm{d}s$$
$$+ \int_{S\setminus x_b} [(\partial_n \mathbf{v}\cdot\mathbf{n}_S)\vartheta + \partial_{tg}((\mathbf{v}\cdot\mathbf{t}_S)\vartheta)]\delta S\,\mathrm{d}s$$
$$+ [j(P)]_{x_b}\frac{\mathbf{n}_S\cdot\mathbf{n}_\Sigma}{\mathbf{n}_S\cdot\mathbf{t}_\Sigma}\delta S(x_b) - \mathbf{t}\cdot\left[\frac{\partial j}{\partial \mathbf{n}_S}\right]_{x_b}\delta S(x_b)$$

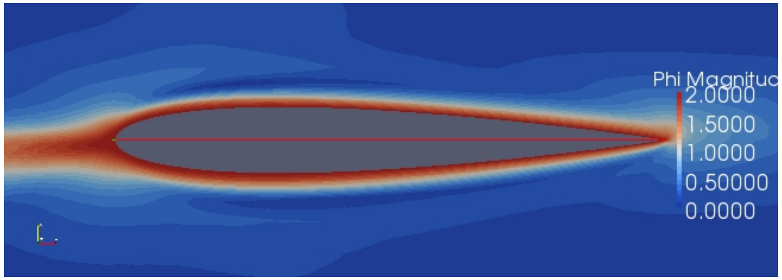
$$\begin{cases} -\mathbf{A}^T\cdot\nabla\Psi = 0, & \text{in } \Omega\setminus\Sigma \\ \boldsymbol{\varphi}\cdot\mathbf{n}_S = \frac{\partial j}{\partial U}, & \text{on } S\setminus x_b \\ \Psi^T(\mathbf{A}\cdot\mathbf{n}_{\Gamma_\infty})_- = 0, & \text{on } \Gamma_\infty \\ [\Psi^T]_\Sigma = 0, & \text{on } \Sigma \\ \partial_{tg}\Psi^T[\mathbf{F}\cdot\mathbf{t}_\Sigma] = 0, & \text{on } \Sigma \\ \Psi^T(x_b)[\mathbf{F}\cdot\mathbf{t}_\Sigma]_{x_b} = \frac{[j]_{x_b}}{n_S\cdot\mathbf{t}_\Sigma}, & \text{at } x_b \\ L = \Psi|_\Sigma, & \text{on } \Sigma \end{cases}$$




## Surface sensitivity Navier-Stokes' case

$$\delta J = \delta \int_S j\left(\vec{f}, T\right)\,dS = \int_{\delta S} j\left(\vec{f}, T\right)\,ds - I_{eq},$$
$$I_{eq} = \int_S \left(\vec{n}_S\cdot\delta\vec{v}\,(\rho\psi_1 + \rho H\psi_4) - \psi_4\vec{n}_S\cdot\sigma\cdot\delta\vec{v}\right)ds$$
$$+ \int_S \left(\vec{n}_S\cdot\Sigma\cdot\delta\vec{v} - k\psi_4\partial_n(\delta T)\right)ds,$$

$$\begin{cases} -\left(\vec{A} - \vec{A}^v\right)^T\cdot\vec{\nabla}\Psi + \vec{\nabla}\cdot\left(\left(\begin{array}{cc} D^T_{xx} & D^T_{xy} \\ D^T_{yx} & D^T_{yy}\end{array}\right)\cdot\left(\begin{array}{c}\partial_x\Psi \\ \partial_y\Psi\end{array}\right)\right) = 0. \\ \Psi_2 = \frac{\partial j}{\partial f_x}, \quad \Psi_3 = \frac{\partial j}{\partial f_y}, \\ \quad k\partial_n\Psi_4 = \frac{\partial j}{\partial T}. \end{cases}$$



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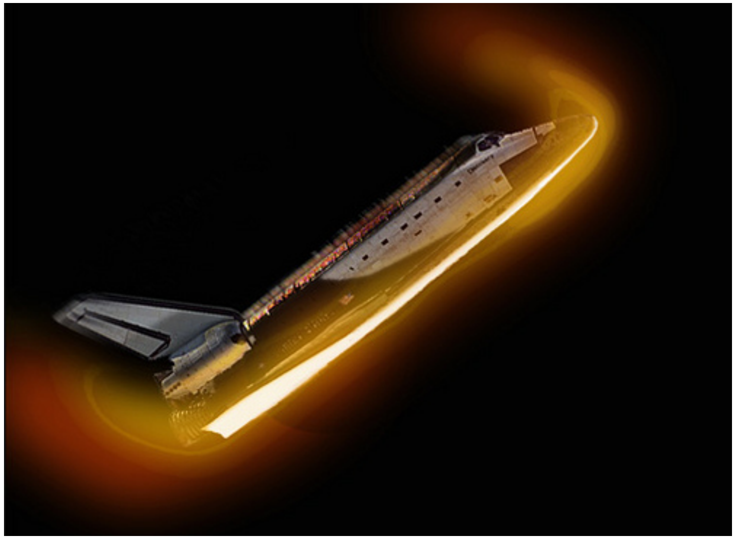
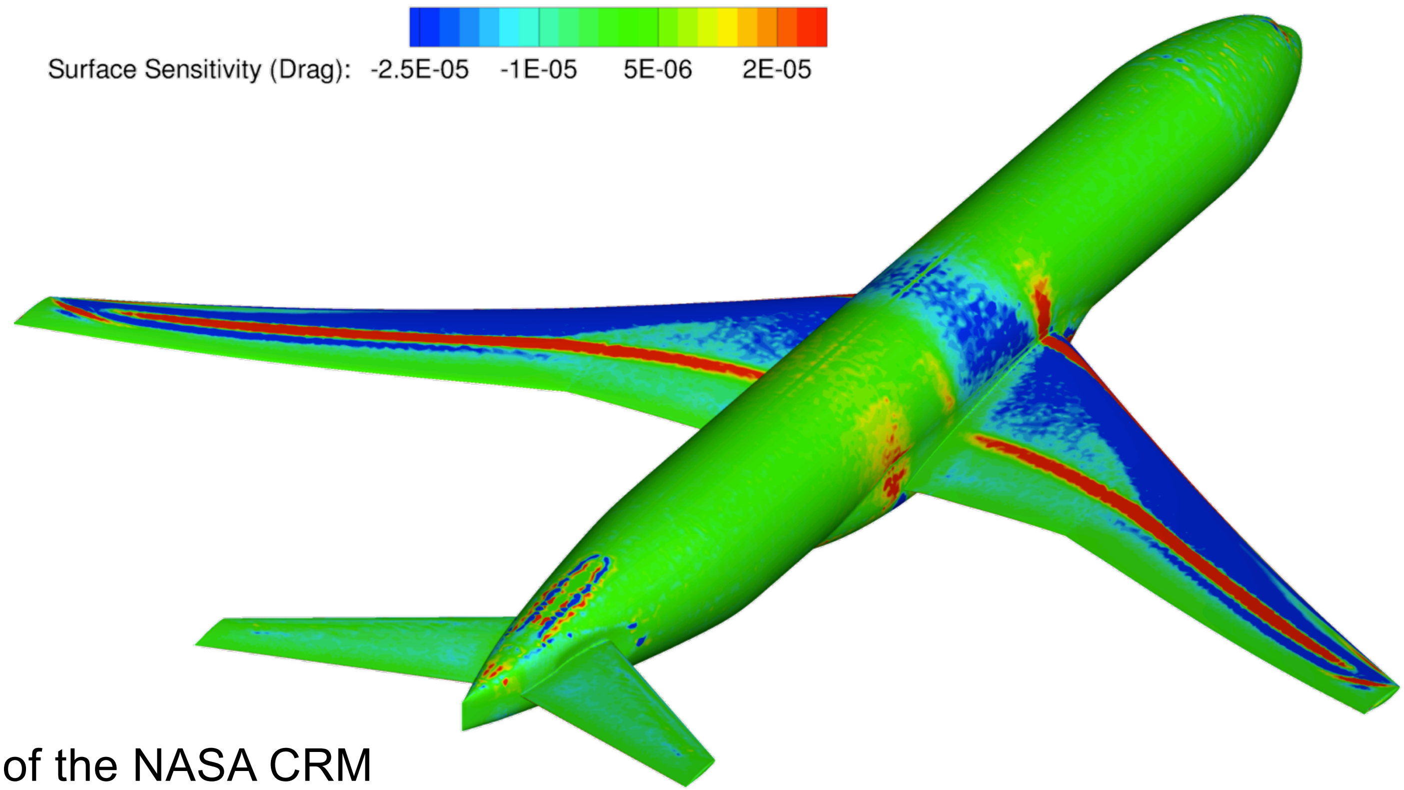


Image | UCI Flight Dynamics & Control Lab

# Large transonic RANS design with no memory or time overhead.




Redesign of the NASA CRM configuration (DPW-5):

- Mach number 0.85.
- Reynolds number $5.10^6$.
- Angle of attack 2.37deg.
- Hybrid grid with $10\ 10^6$ elements, RANS equations and SA turbulence model.

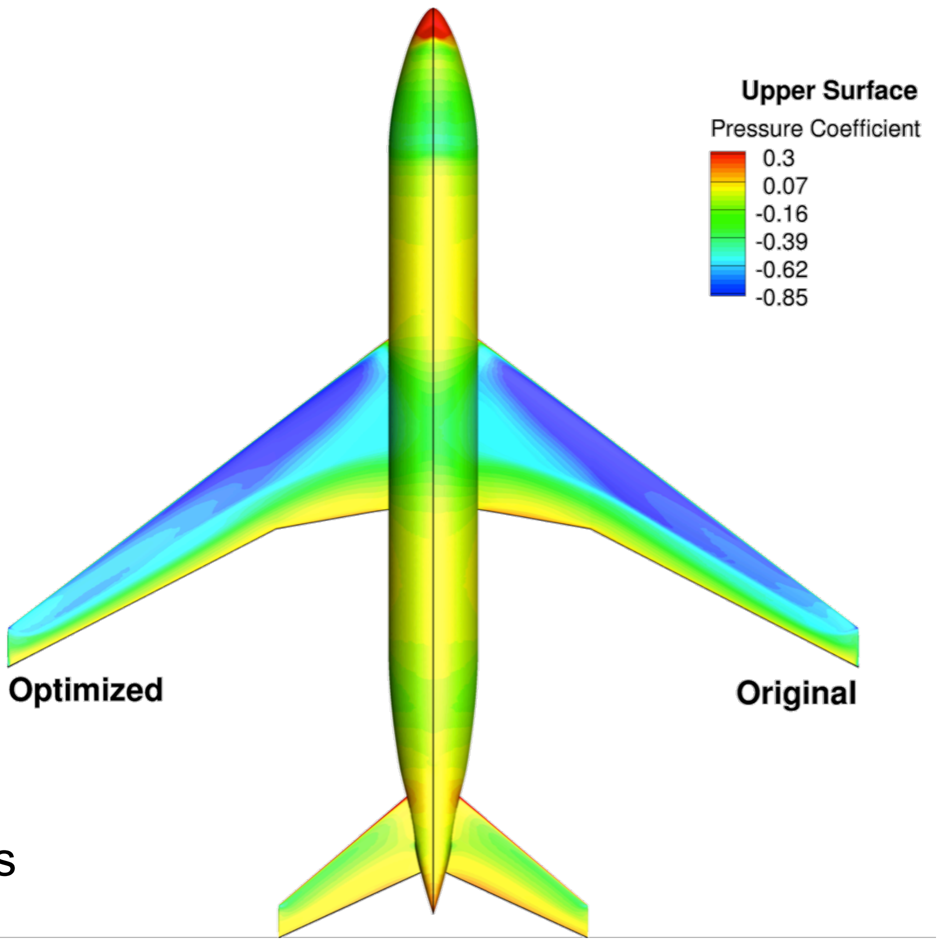
# Large transonic RANS design with no memory or time overhead.

Optimization problem description:

- $C_D$ minimization.
- Maintaining $C_L$ and $C_{My}$.
- And 95% of the original thickness in 5 sections.

Optimization problem result:

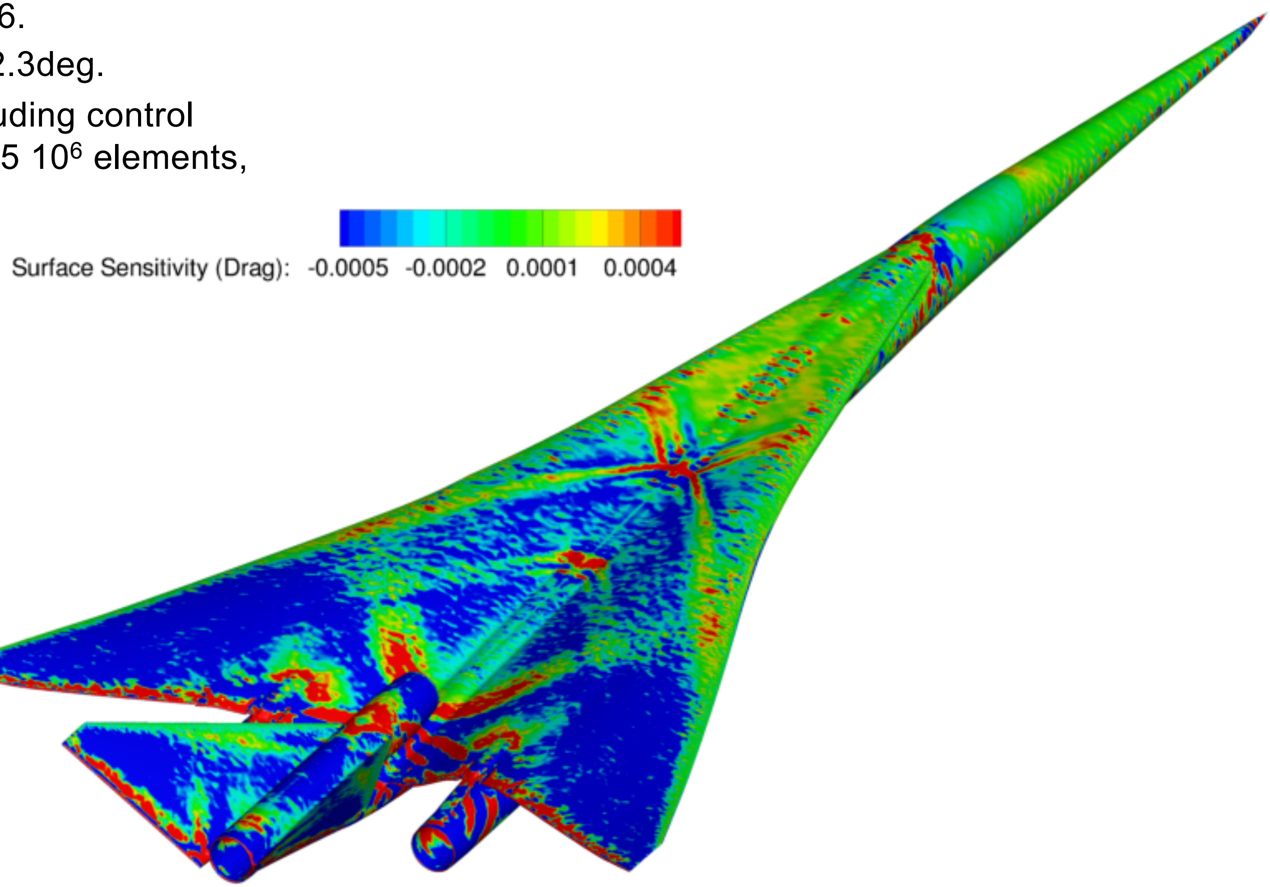
- The final design (optimizer iteration 15) satisfied $C_L$ and thickness constraints (1.5% reduction in $C_L$)
- with a smaller $C_D$ (2.2% reduction, or 7 drag counts less than the baseline value).



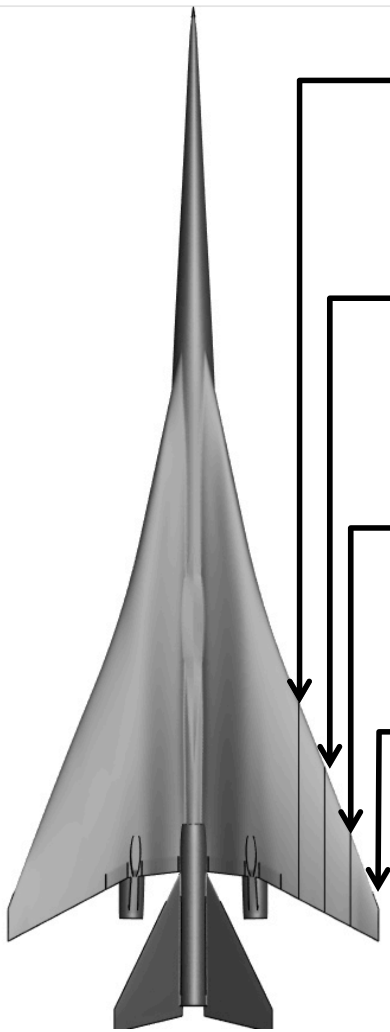
# Highly detailed/complex geometries with efficient numerical schemes.

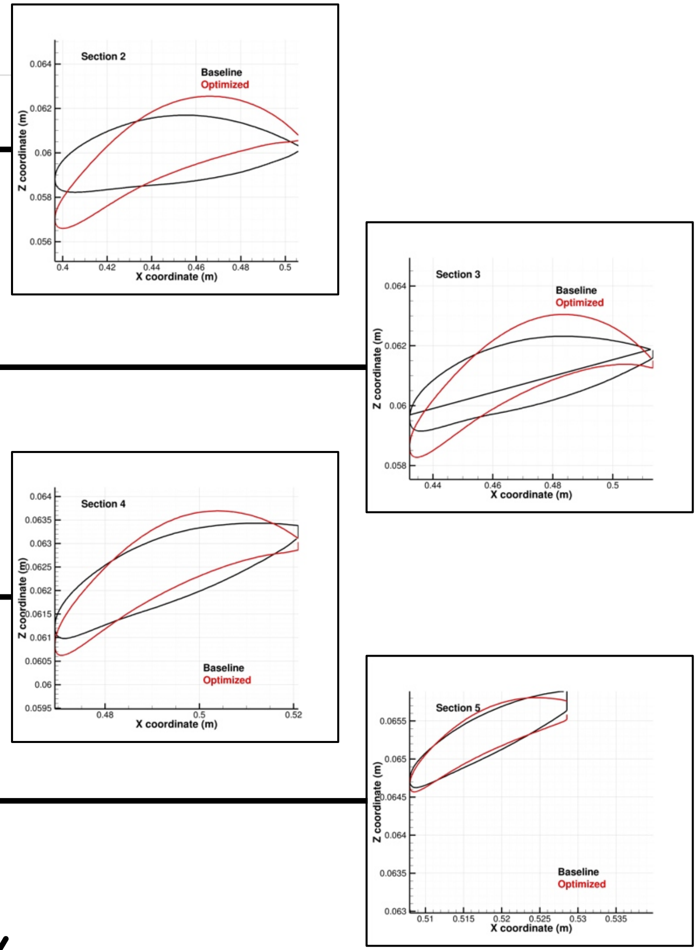
Redesign of the Lockheed Martin 1021 configuration (SBPW-1):

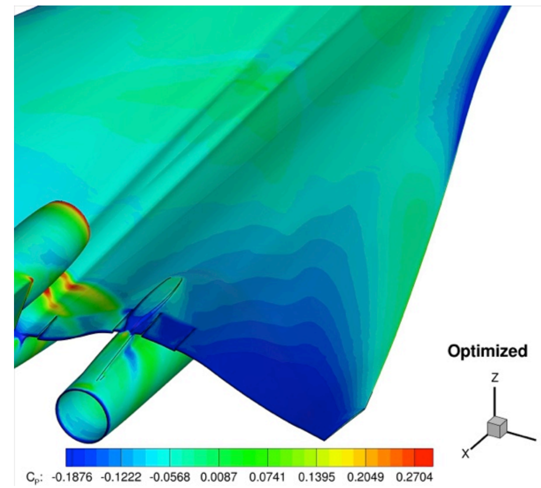
- Mach number 1.6.
- Angle of attack 2.3deg.
- Hybrid grid (including control surfaces) with 7.5 $10^6$ elements, Euler equations.

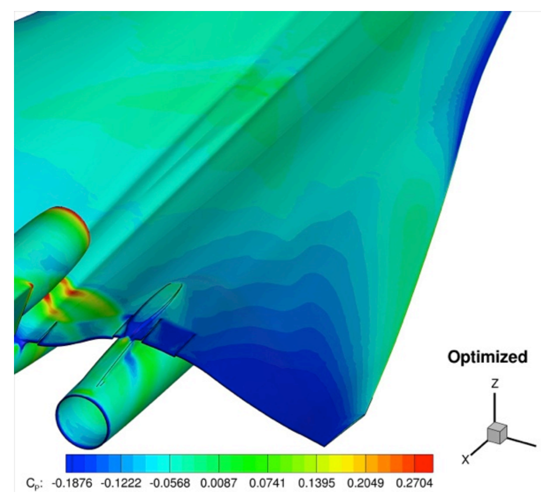


# Highly detailed/complex geometries with efficient numerical schemes.






- Decrease the pitching moment by 25%
- Maintain 95% of the original lift.
- Minimum twist angle (-3.3deg).
- Maintain 90% of the original maximum thicknesses at 5 wing sections.

# Complex governing equations (non-equilibrium flows)

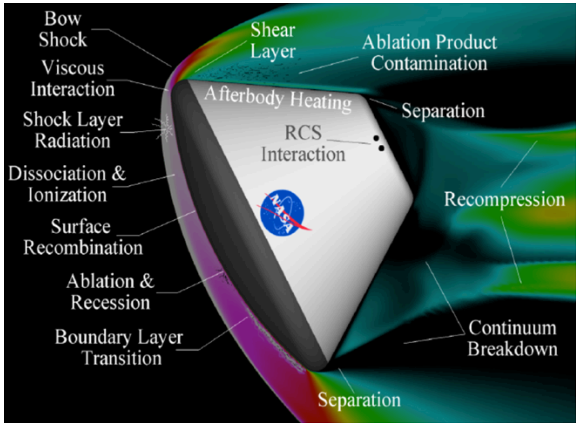



Figure: Physics of the nonequilibrium environment. Courtesy NASA

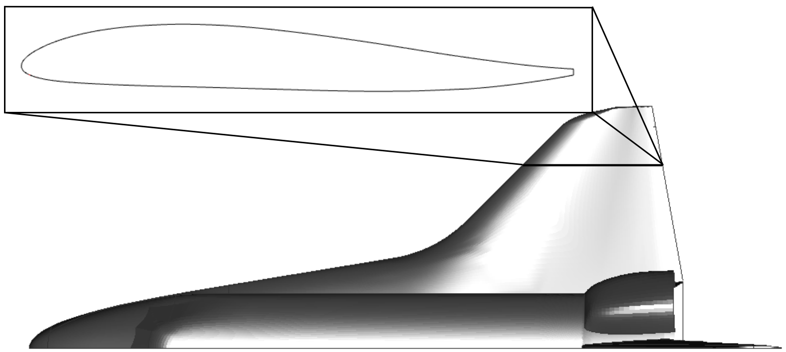
## Problem description

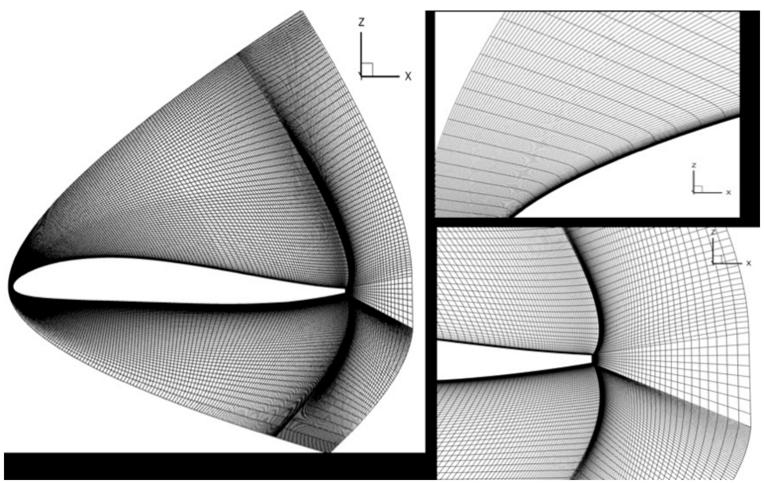
- Unique design environment
- Complex, interacting phenomena
- Physics @ different time & length scales
- Challenging & expensive to simulate
- Ground facilities unable to completely replicate flight environment
- Safety & mission critical

## Modelization

- Steady, viscous, reactive gas mixture in thermochemical non-equilibrium
- Single fluid, two-temperature model
- Finite-rate chemistry
- Rigid-Rotator Harmonic Oscillator (RRHO) thermodynamics
- Landau-teller vibrational relaxation model

# Complex governing equations (non-equilibrium flows)





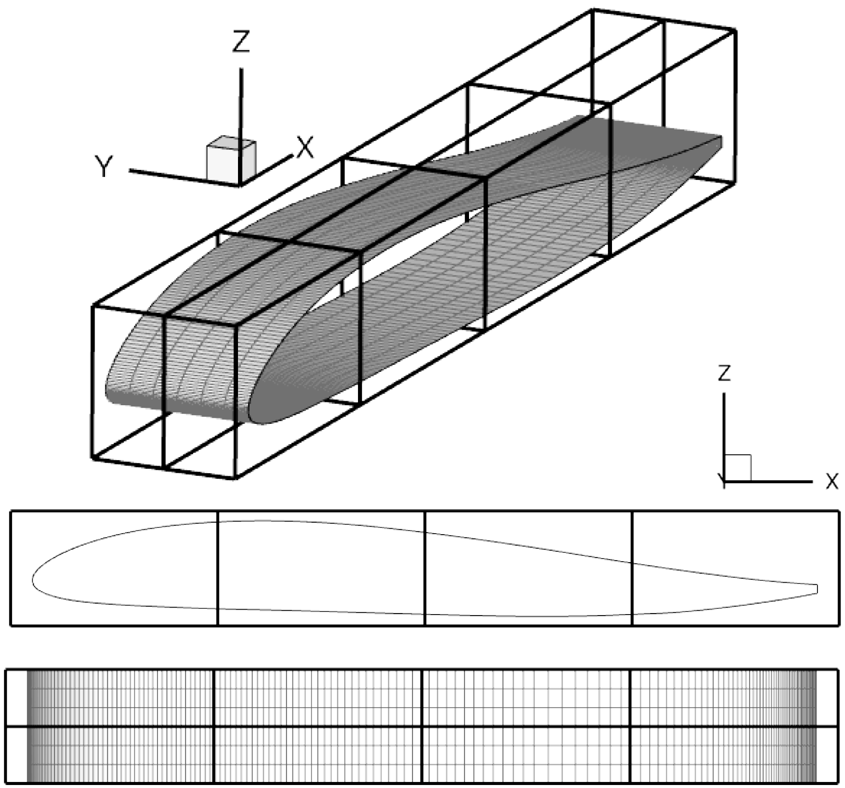

Why this is an interesting problem for the continuous adjoint methodology?

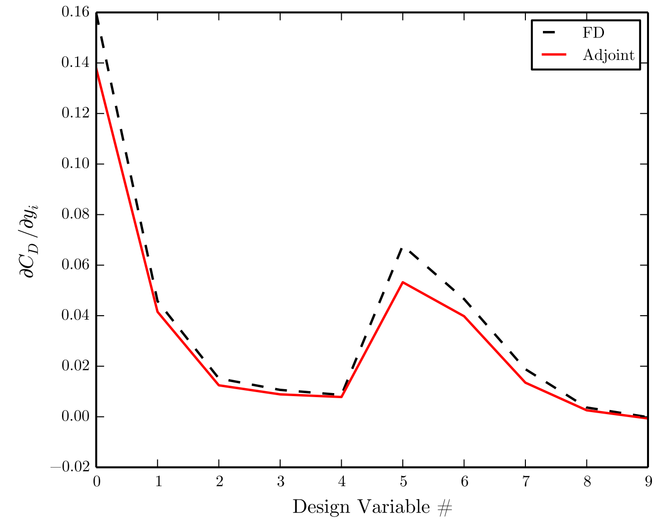
- Expensive simulations
- Previous experiences. Connection and arrangement of the differentiated routines to compute the discrete adjoint equation. The resulting problem is highly ill-conditioned (close to singular) can we really trust the final result?
- Surface formulation.
- Numerical methods are critical in the direct problem...
- The possibility of understanding the physic.

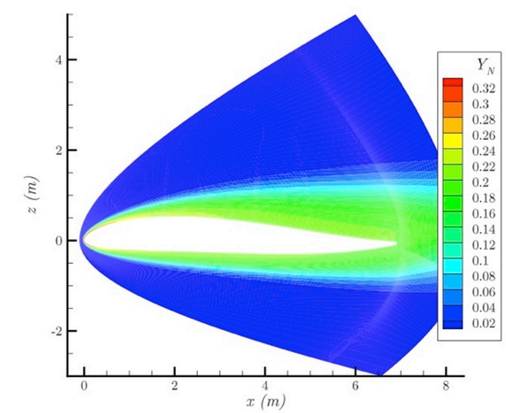
Figure: Artist concept of shuttle entry. Courtesy UCI Flight Dynamics & Control Lab

From S. Copeland's PhD Thesis

# Complex governing equations (non-equilibrium flows)




$$\frac{\partial \mathcal{J}}{\partial S} = -\vartheta \delta \vec{u} \cdot \vec{n} - \sum_s \psi_{\rho_s} \left(\delta \vec{J}_s \cdot \vec{n}\right) - \psi_{\rho e} \sum_s \left(\delta \vec{J}_s \cdot \vec{n}\right) h_s^{int} - \psi_{\rho e^{ve}} \sum_s \left(\delta \vec{J}_s \cdot \vec{n}\right) e_s^{ve}$$
$$+ \psi_{\rho e} \bar{\bar{\sigma}} \cdot \delta \vec{u} \cdot \vec{n} + \psi_{\rho e} \delta(\kappa^{tr} \nabla T) \cdot \vec{n} + ((\psi_{\rho e} + \psi_{\rho e^{ve}}) \delta(\kappa^{ve} \nabla T^{ve}) \cdot \vec{n}) - \left(\mu(\bar{\bar{\Sigma}}^{\phi} \cdot \vec{n}) \delta \vec{u}\right).$$

From S. Copeland's PhD Thesis

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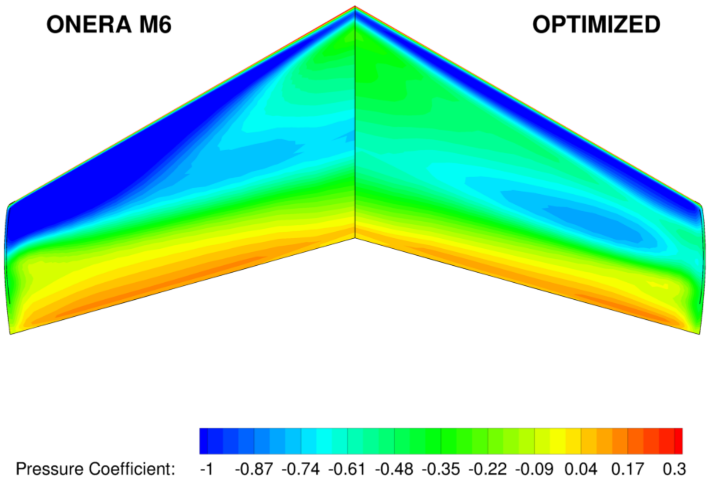
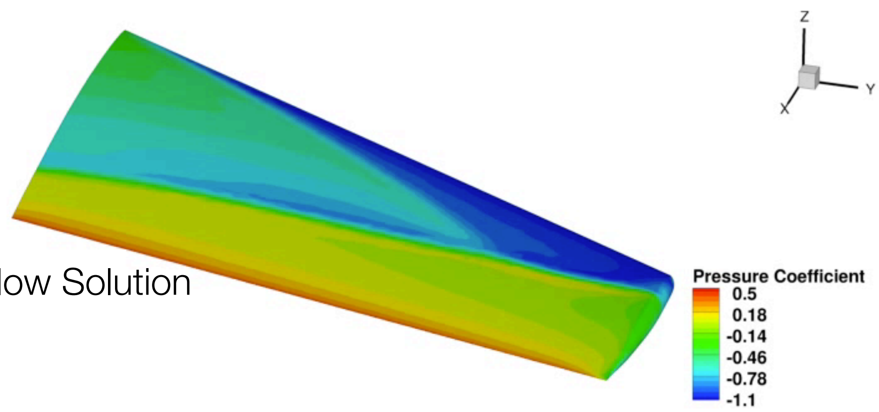


# Unsteady aerodynamic shape design

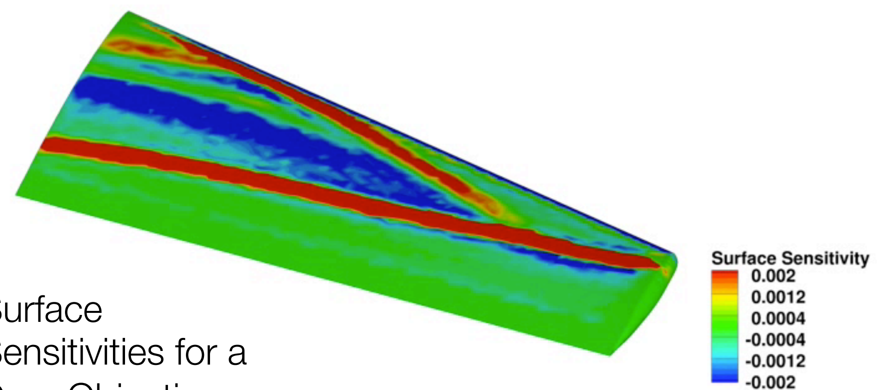
- Challenges for unsteady design:
  - Computational cost and memory can increase dramatically for time-accurate simulations
  - Handling moving surfaces / dynamic meshes in the formulation requires the Arbitrary Lagrangian-Eulerian (ALE) form of the flow equations.

- Due to the complexity of the unsteady design problem, a continuous adjoint approach is appealing due to…
  - Recovering a surface formulation for the gradient with no dependence on the volume mesh.
  - Flexibility in numerical methods to help mitigate convergence issues for stiff problems
  - The time-accurate, continuous adjoint PDE can also be discretized for different problems immediately (rotating frame or time-spectral approaches for instance)

# Unsteady aerodynamic shape design




Flow Solution



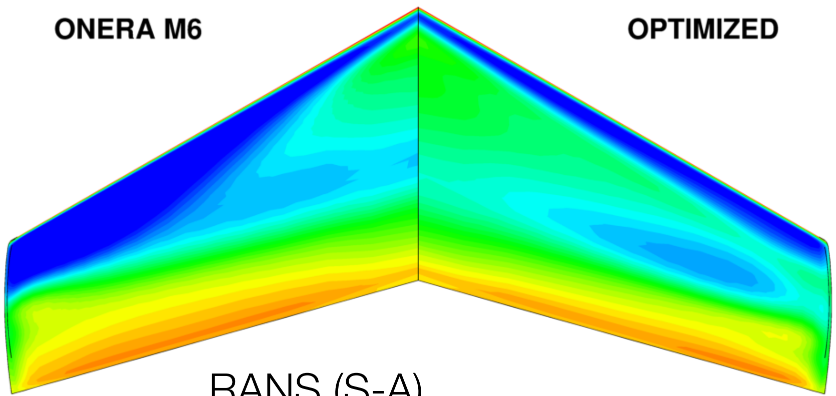

Surface Sensitivities for a Drag Objective

- Pitching wing in transonic flow:
  - Mach = 0.8395
  - Reduced frequency of 0.1682
  - Mean alpha of 3.06 degrees
  - Pitching amplitude of 2.5 degrees
  - Reynolds number = 11.72 million
- Pitching about the y-axis through the root quarter-chord
- 25 time steps per period for 7 periods.
- Unsteady RANS (S-A) equations on rigidly transforming meshes

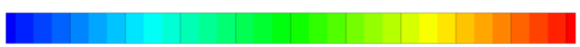
From T. Economon's PhD Thesis

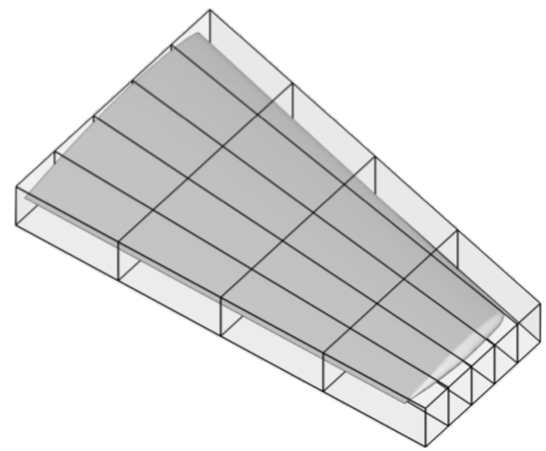
# Unsteady aerodynamic shape design
## ONERA M6

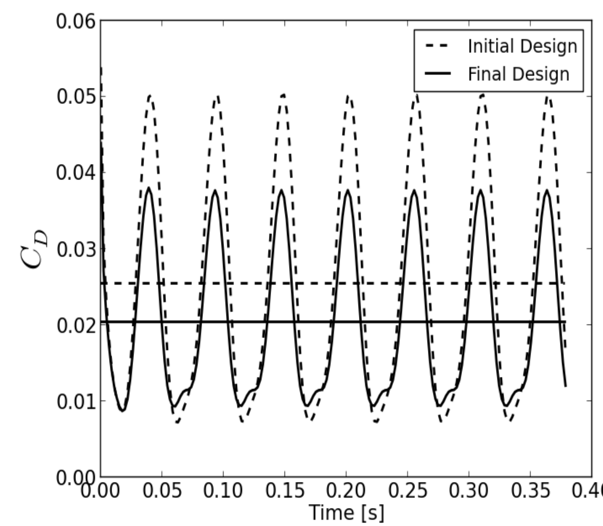
25 upper surface control point variables




RANS (S-A)
(incidence of max drag)






20.3 % Reduction in Time-averaged Drag

$$
\begin{aligned}
\delta\mathcal{J} = \frac{1}{\mathbb{T}}\int_{t_o}^{t_f}\int_S & \{\vec{d}\cdot[\vec{q}_{\rho\vec{v}} - \partial_t(\rho\vec{v})] + \nabla\vec{d} : (\bar{\bar{I}}p - \bar{\bar{\sigma}}) - (\bar{\bar{I}}p - \bar{\bar{\sigma}})\cdot\vec{n}\cdot\partial_n\vec{d} + \vartheta\partial_n(\vec{v} - \vec{u}_\Omega)\cdot\vec{n} \\
& - \psi_{\rho E}\partial_n(\vec{v} - \vec{u}_\Omega)\cdot\bar{\bar{\sigma}}\cdot\vec{n} + \vec{n}\cdot\left(\bar{\bar{\Sigma}}^{\varphi} + \bar{\bar{\Sigma}}^{\psi_{\rho E}}\right)\cdot\partial_n(\vec{v} - \vec{u}_\Omega) - \mu_{tot}^2 c_p \nabla_S(\psi_{\rho E})\cdot\nabla_S(T) \\
& - \psi_{\rho E}\left[p(\nabla\cdot\vec{v}) - \bar{\bar{\sigma}} : \nabla\vec{v} + \partial_t(\rho E) + (\vec{q}_{\rho\vec{v}} - \partial_t(\rho\vec{v}))\cdot\vec{v} - q_{\rho E}\right]\}\delta S\, ds\, dt, \\
= \frac{1}{\mathbb{T}}\int_{t_o}^{t_f}\int_S & \left\{\frac{\partial\mathcal{J}}{\partial S}\right\}\delta S\, ds\, dt
\end{aligned}
$$

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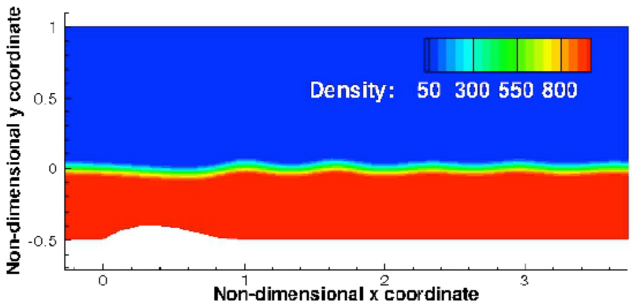
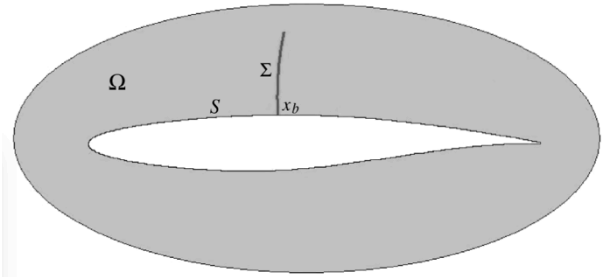
CONCLUSIONS



# Everything starts with a very simple observation

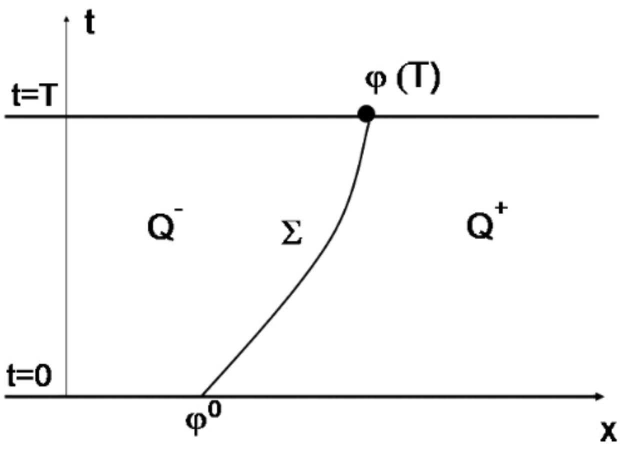
In the inviscid case, the simple and "natural" rule. Breaks down in the presence of shocks

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \rightarrow \frac{\partial \delta u}{\partial t} + \delta u \frac{\partial u}{\partial x} + u\frac{\partial \delta u}{\partial x} = 0$$

$$\delta u = \text{discontinuous}, \ \frac{\partial u}{\partial x} = \text{Dirac delta} \Rightarrow \delta u \frac{\partial u}{\partial x} ????$$

The difficulty may be overcome with a detailed analysis of the problem.
A new viewpoint: solution = solution + shock location, The the pair solves:

$$\begin{cases} \partial_t u + \partial_x \left(\frac{u^2}{2}\right) = 0, & \text{in } Q^- \cup Q^+, \\ \varphi'(t)[u]_{\varphi(t)} = [u^2/2]_{\varphi(t)}, & t \in (0,T), \\ \varphi(0) = \varphi^0, & \\ u(x,0) = u^0(x), & \text{in } \{x < \varphi^0\} \cup \{x > \varphi^0\}. \end{cases}$$




The corresponding linearized system is:

$$\begin{cases} \partial_t \delta u + \partial_x (u\delta u) = 0, \quad \text{in } Q^- \cup Q^+, \\ \delta\varphi'(t)[u]_{\varphi(t)} + \delta\varphi(t)\left(\varphi'(t)[u_x]_{\varphi(t)} - [u_x u]_{\varphi(t)}\right) \\ \qquad + \varphi'(t)[\delta u]_{\varphi(t)} - [u\delta u]_{\varphi(t)} = 0, \quad \text{in } (0,T), \\ \delta u(x,0) = \delta u^0, \quad \text{in } \{x < \varphi^0\} \cup \{x > \varphi^0\}, \\ \delta\varphi(0) = \delta\varphi^0, \end{cases}$$

# Transonic airfoil design

Is this really useful in practical applications?

- In the presence of shock waves the mathematical formulation requires a difficult nonstandard analysis.
- In simpler models, this formulation improves the performance of the optimization process.

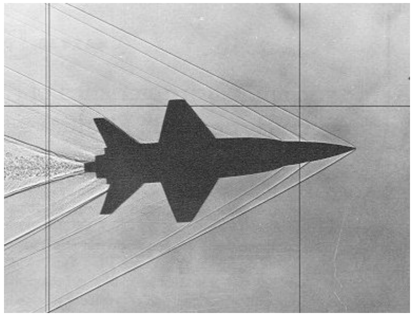
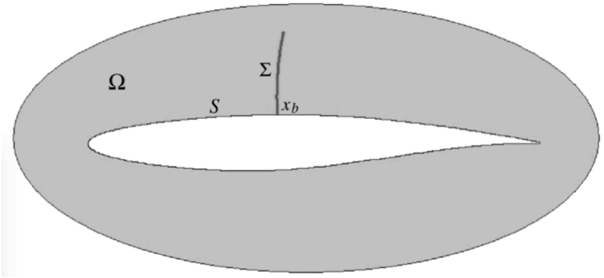


Image | X-15 aircraft at Mach 3.5 via NASA webpage

$$
\begin{cases}
-\vec{A}^T \cdot \vec{\nabla}\Psi = 0, & \text{in } \Omega \setminus \Sigma, \\
\vec{\varphi} \cdot \vec{n}_S = \frac{\partial j}{\partial U}, & \text{on } S \setminus x_b, \\
\Psi^T (\vec{A} \cdot \vec{n}_{\Gamma_\infty})_- = 0, & \text{on } \Gamma_\infty, \\
\left[\Psi^T\right]_\Sigma = 0, \quad \partial_{tg}\Psi^T \left[\vec{F} \cdot \vec{t}_\Sigma\right] = 0, & \text{on } \Sigma, \\
\Psi^T (x_b) \left[\vec{F} \cdot \vec{t}_\Sigma\right]_{x_b} = \frac{[j]_{x_b}}{n_S \cdot \vec{t}_\Sigma}, & \text{at } x_b.
\end{cases}
$$

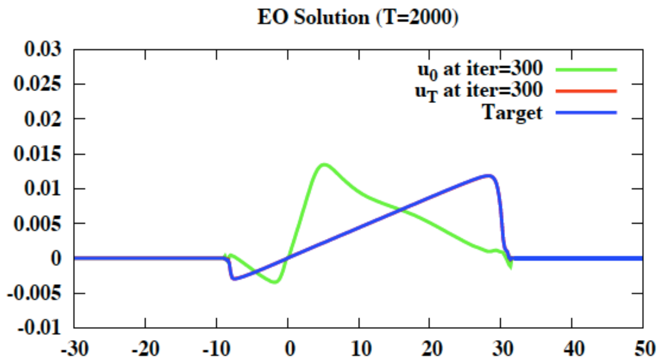


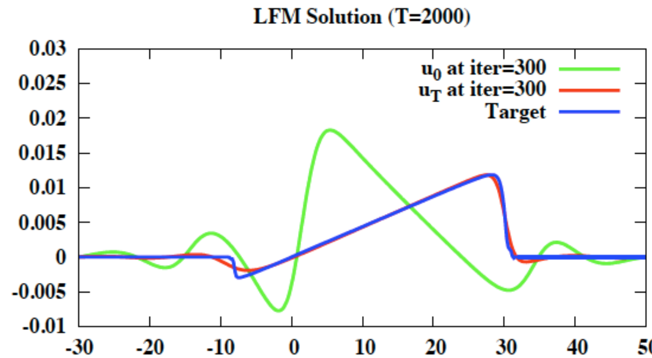

What is the most efficient way to move the shock ahead the crest?

$$
\begin{cases}
\delta J(S) = \int_S \left[ \frac{\partial j}{\partial P} \partial_n P + \vec{t} \cdot \partial_{tg} \left( \frac{\partial j}{\partial \vec{n}_S} \right) - \kappa \left( j + \frac{\partial j}{\partial \vec{n}_S} \vec{n}_S \right) \right] \delta S \, ds \\
+ \int_{S \setminus x_b} \left[ (\partial_n \vec{v} \cdot \vec{n}_S) \vartheta + \partial_{tg} \left( (\vec{v} \cdot \vec{t}_S) \vartheta \right) \right] \delta S \, ds - [j(P)]_{x_b} \frac{\vec{n}_S \cdot \vec{n}_\Sigma}{\vec{n}_S \cdot \vec{t}_\Sigma} \delta S (x_b)
\end{cases}
$$

# Inverse design (wave propagation)

$$\frac{\partial P}{\partial \sigma} = P\frac{\partial P}{\partial \tau} + \underbrace{\frac{1}{\Gamma}\frac{\partial^2 P}{\partial^2 \tau}}_{\text{absorption}} + \underbrace{\sum_{\nu} C_{\nu} \frac{\frac{\partial^2}{\partial \tau^2}}{1+\theta_{\nu}\frac{\partial}{\partial \tau}} P}_{\text{molecular relaxations}} - \underbrace{\frac{1}{2G}\frac{\partial G}{\partial \sigma} P}_{\text{ray-tube spreading}} + \underbrace{\frac{1}{2\rho_0 c_0}\frac{\partial(\rho_0 c_0)}{\partial \sigma} P}_{\text{atmospheric stratification}}$$

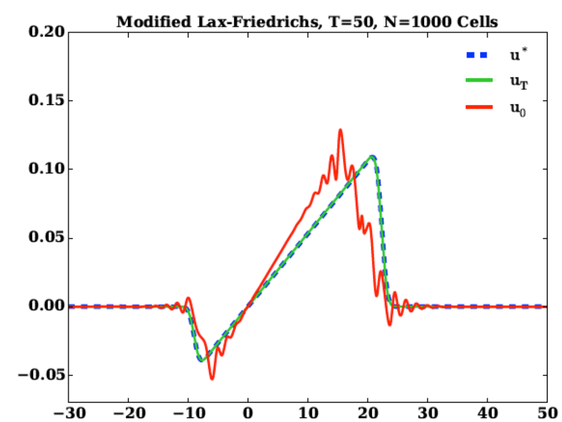



The solution by Engquist-Osher recovers successfully the target, while the one by Lax-Friedrichs needs to introduce spurious oscillations.

What happens if we use a nonlinear optimization package (IPOPT) instead of a gradient descent methods (GDM)?

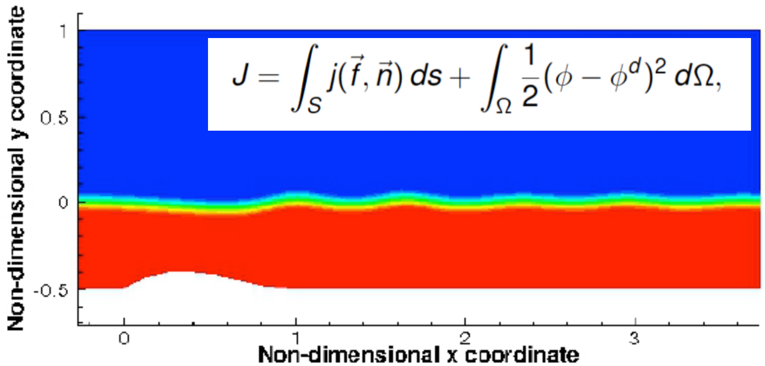
IPOPT runs are more computationally intensive due to the fact that IPOPT assembles and solves a linear system for the solution of Karush-Kuhn-Tucker (KKT) optimality condition.



# Free-surface problems

Free-surface design or total resistance minimization Fundamental in problems where the target is:

- Reduce the wave energy (ship design).
- Increase the size of the wave (surfing wave pools)




The following set of adjoint equations have to be solved to compute the variation of the functional:

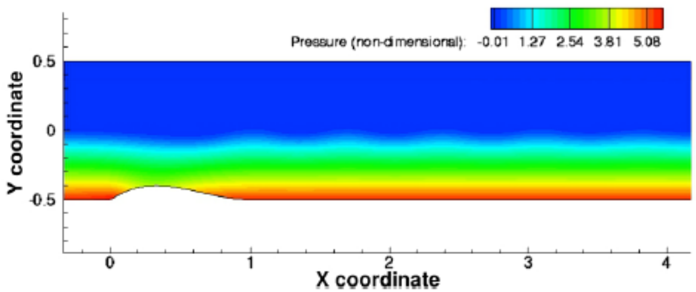
$$\begin{cases} -\vec{A}_U^T \cdot \vec{\nabla}\Psi_U - \vec{\nabla}((\vec{\nabla}\Psi_U^T)\mathbf{D}_U = \frac{\phi}{\rho}\left(0 \;\; \partial_x\psi_\phi \;\; \partial_y\psi_\phi \;\; \partial_z\psi_\phi\right)^T & \text{in } \Omega, \\ -\vec{v} \cdot \vec{\nabla}\psi_\phi = \left(\vec{\nabla}\Psi_U^T \frac{\partial \vec{F}^c}{\partial \rho} - \frac{\phi \vec{\nabla}\Psi_\phi \vec{v}}{\rho} - \frac{\Psi_4}{Fr^2}\right)\frac{\partial \rho}{\partial \phi} - \vec{\nabla}\Psi_U^T \frac{\partial \vec{F}^v}{\partial \mu}\frac{\partial \mu}{\partial \phi} + (\phi - \phi^d) & \text{in } \Omega, \\ \vec{\varphi} = \vec{d} & \text{on } S, \end{cases}$$

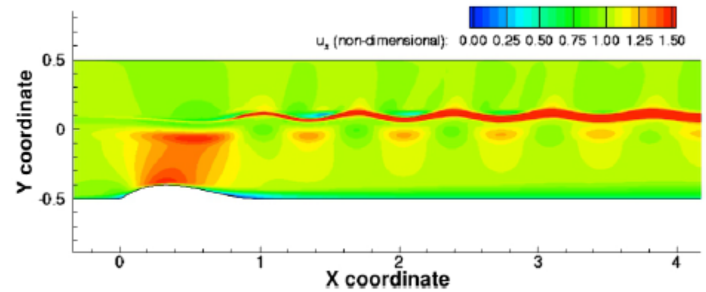
The variation of the functional is computed as

$$\delta J = \int_S \left(\frac{1}{2}(\phi - \phi^d)^2 - (\vec{n} \cdot \partial_n \vec{v})\left(\frac{\beta^2}{\rho}\psi_1 + \psi_\phi \phi\right) + \vec{n} \cdot \bar{\tau}^\varphi \cdot \partial_n \vec{v}\right)\delta S\, ds$$

It is possible to do inverse design of the interface between two fluids!

# Free-surface problems




The order of magnitude of both shape sensitivities is completely different. The shape of the free-surface is very insensitive to changes in the bottom of the channel.

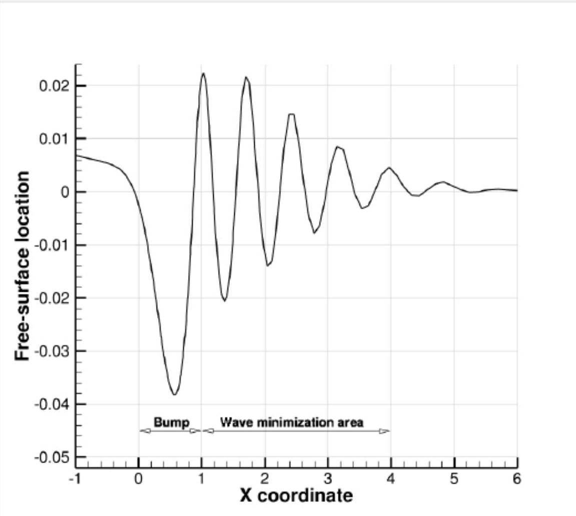



Figure : Diagram of the free-surface sensitivity problem.

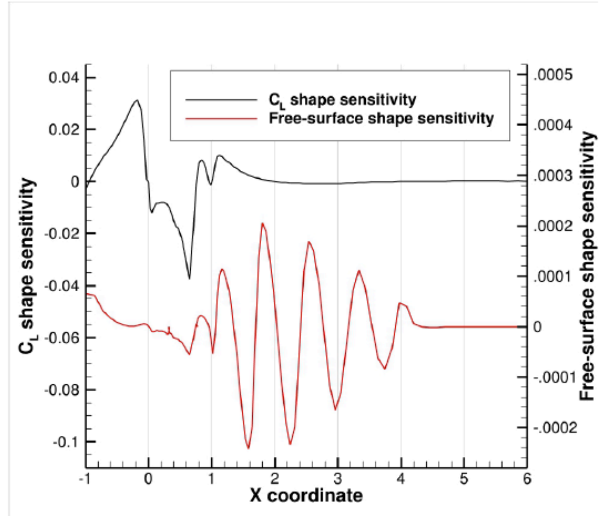



Figure : Shape sensitivity for free-surface, and lift design problems.

INTRODUCTION

SURFACE FORMULATION

COMPLEX SYSTEMS

UNSTEADY DESIGN

DESIGN WITH DISCONTINUITIES

**CONCLUSIONS**

# Conclusions

Aerodynamic Optimal Shape Design, created and developed by Prof. Jameson, is a **high-impact field with a need for sophisticated engineering, mathematical and computational developments.**

- Prof. Jameson legacy is an example about how the **intelligence, enthusiasm and perseverance make miracles**. With very limited computational resources he did some simulations that even today are difficult to repeat. Demonstrating that, from the point of view of success, what is expensive, requires time, patience and spirit are not the instruments but the ability to develop and mature an exemplary aptitude.
- In the internet era it is “easy” to find people with an empty knowledge: they know a lot of information that you can easily find in 5 second using your smartphone. But, **Prof. Jameson knowledge is completely different and impossible to find in other places.**
- The intelligence loves to learn, discover, and create new things. But, **to achieve its owns objectives the personal intelligence should collaborate with the objectives of other people**. The result of this collaboration is a real school of CFD, all inspired by Prof. Jameson generosity and talent.

# The continuous adjoint strikes back

AJ80TH

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