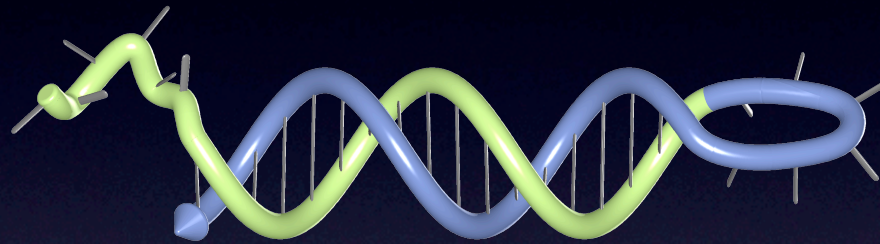


# Molecular Instruments



Niles A. Pierce

California Institute of Technology

In Honor of Antony Jameson

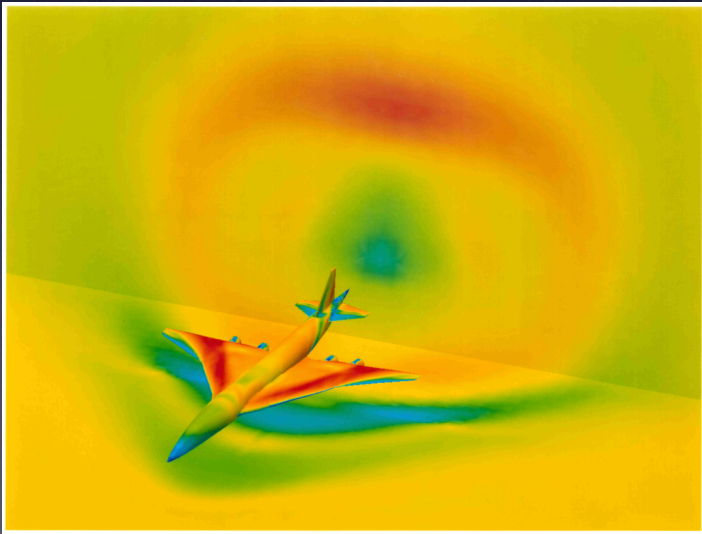
Stanford University

November 21, 2014

# Antony Jameson

## Analysis & Design

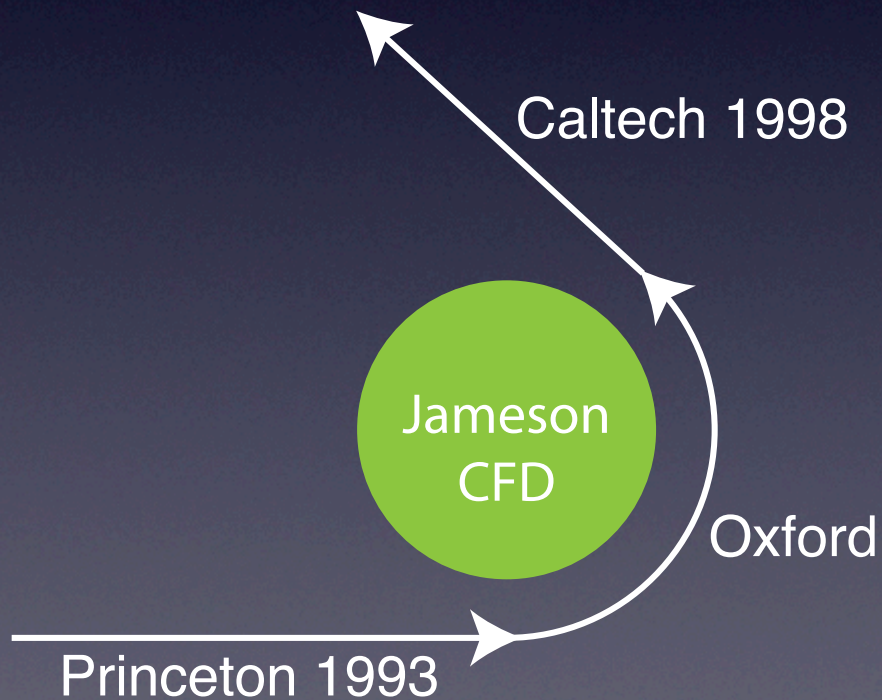
- Defining challenges
- Creative, elegant, practical solutions
- Diverse mathematical and algorithmic principles
- Relentless pursuit of efficiency

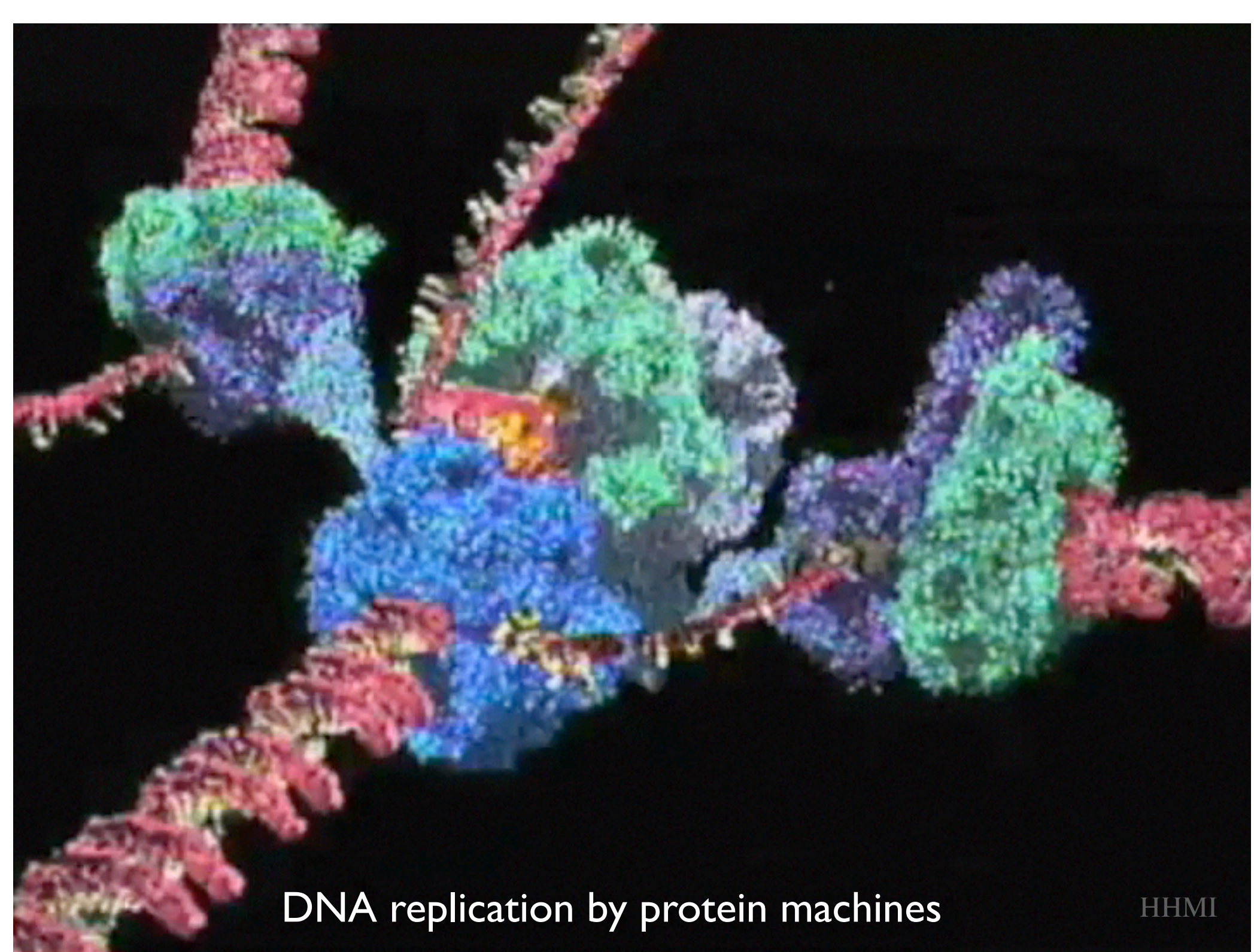


# Antony Jameson

## Analysis & Design

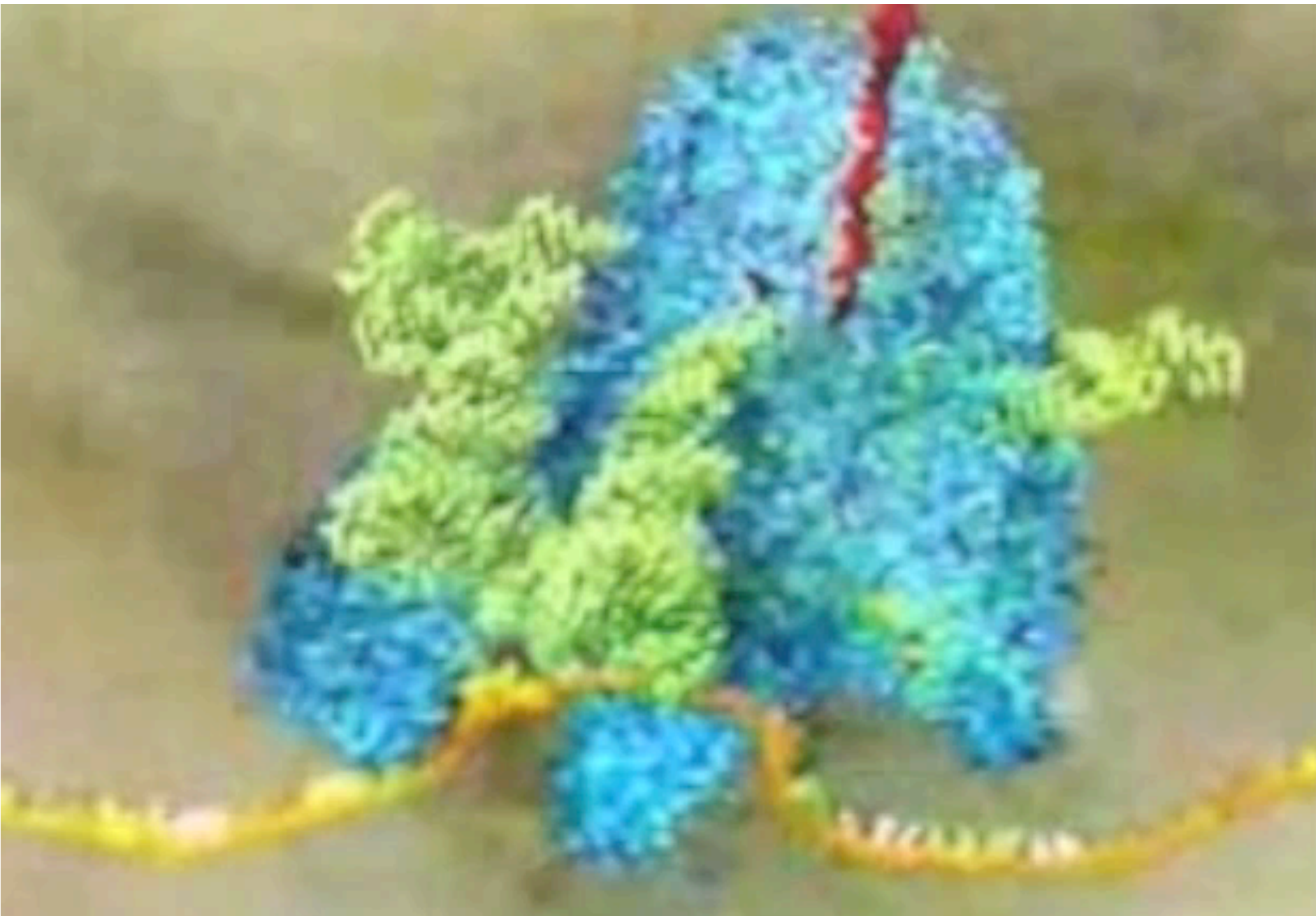
- Defining challenges
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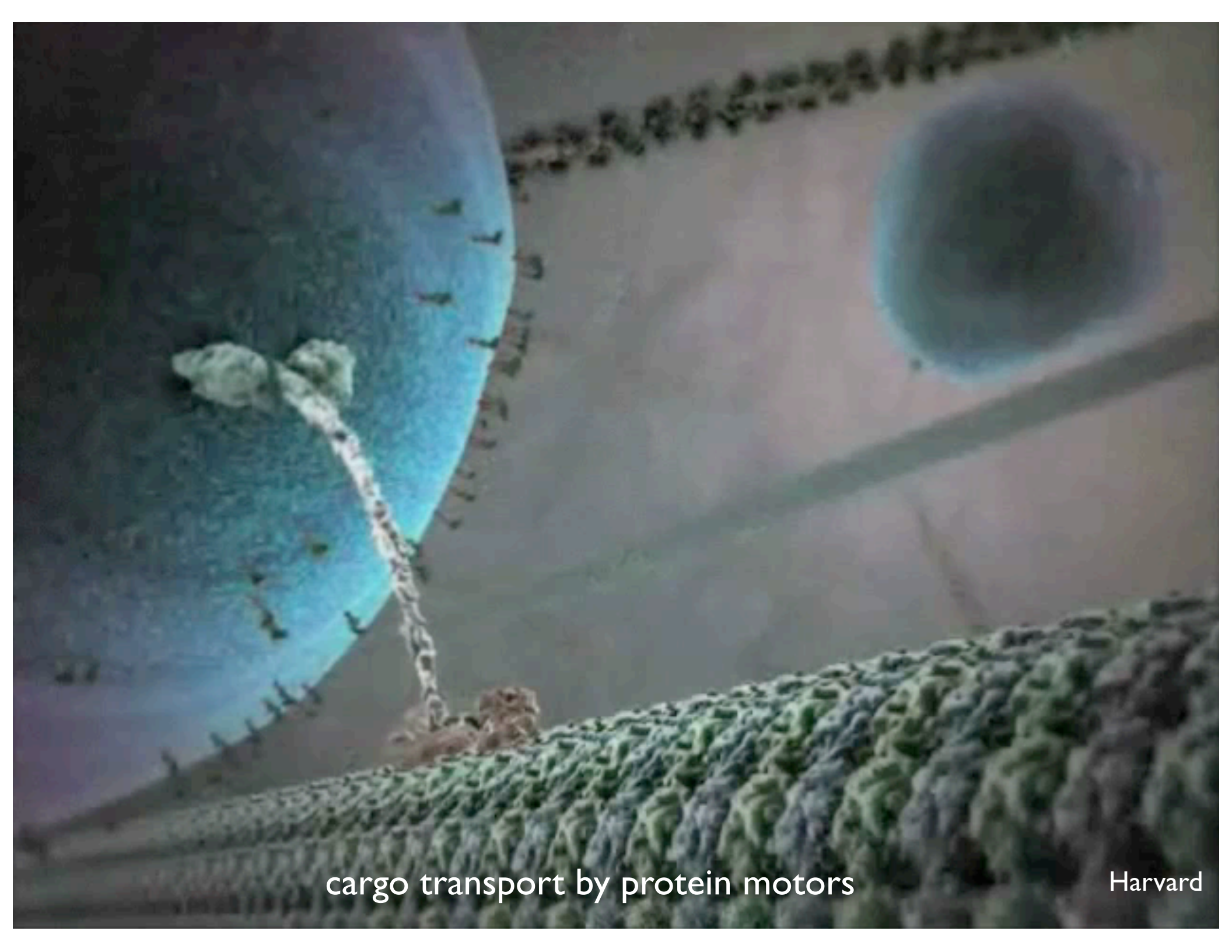
DNA replication by protein machines

HHMI



protein synthesis by RNA and protein machines

HHMI



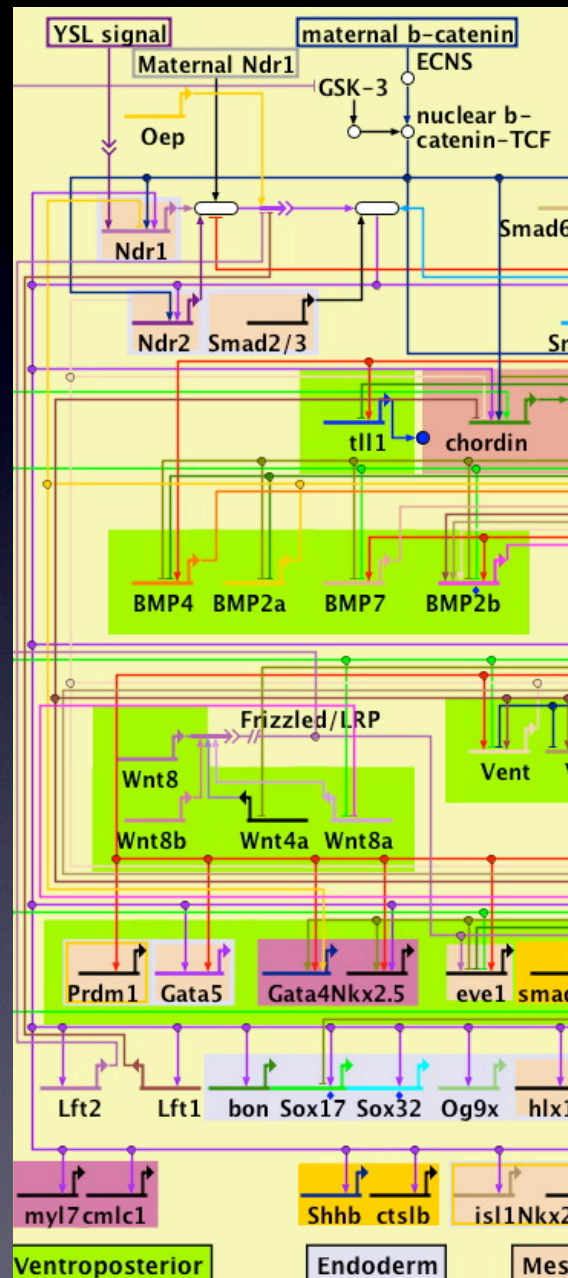
cargo transport by protein motors

Harvard

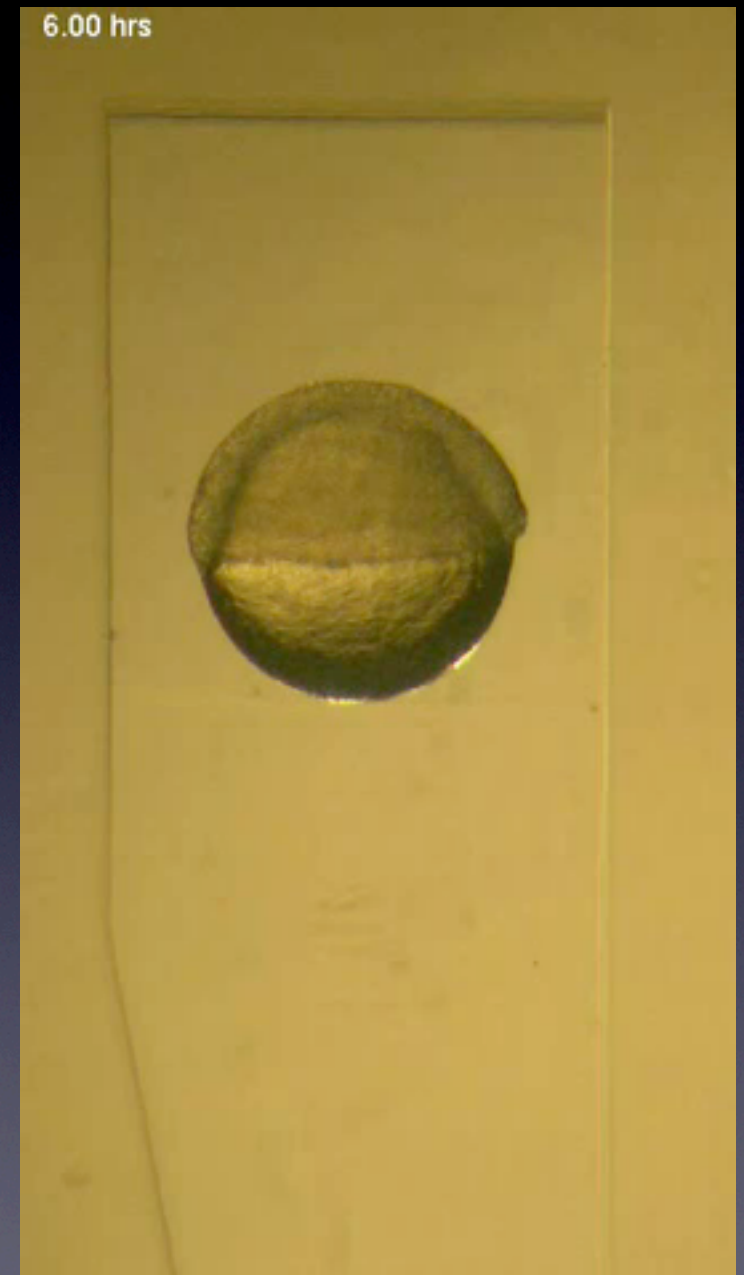
## DNA Genome

...GTGGTACAGGT  
GAATTGGGTAGGC  
TAAATTGTCCATAG  
TTTATGTGTGTGAA  
TGAGGGTGTATGGA  
TGTTTCTCAGAGAT  
GGTTGCAGCTGGA  
AGGGCGTCCATTGT  
GCAAACATATGCT  
GGAGAAGTTGCCGG  
TTCATTCTGCTGTG  
GCGACCCAGATTA  
ATAAAAGGACTAAG  
CCGAAAAGAAAATG  
AACATATATATAT  
ATATATATATATAT  
ATATATATATA...

## Regulatory Circuitry



## Zebrafish Development



Movie: Sean Megason (Harvard)

# Life is orchestrated by programmable biomolecules

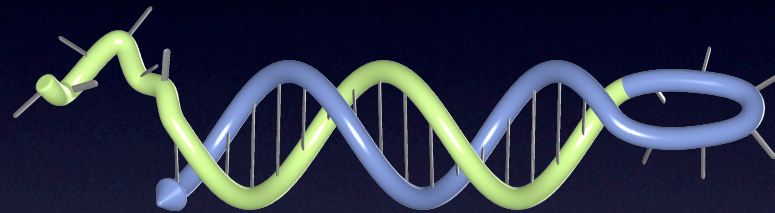
DNA GTGGTACAGGTGAATTTGGGTAGGCTAAATTGTCCATA  
RNA GGCUGUUUUUCGCUGACUUUCAGCCCAAACAAA  
Protein M<sup>T</sup>Y<sup>R</sup>L<sup>E</sup>L<sup>N</sup>G<sup>K</sup>T<sup>L</sup>K<sup>G</sup>R<sup>T</sup>R<sup>T</sup>E<sup>A</sup>V<sup>D</sup>A<sup>A</sup>T<sup>A</sup>E<sup>K</sup>V<sup>F</sup>K<sup>Q</sup>Y<sup>A</sup>N<sup>D</sup>N<sup>G</sup>

Sequencing a genome provides a parts list but no manual

Remains challenging to interrogate and perturb the state of endogenous biological circuitry within intact organisms

# Molecular Instruments

Exploit the very programmability that biological organisms exploit themselves



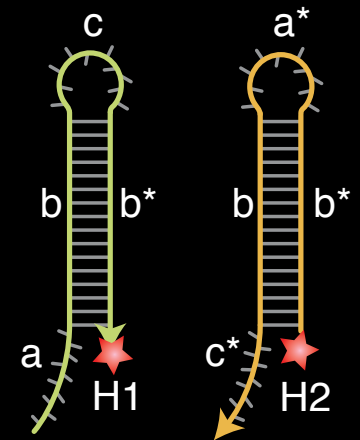
Engineer **small conditional DNAs and RNAs** (scDNAs and scRNAs) that interact and change conformation to read out or regulate the state of endogenous biological circuitry within intact organisms

# Why Nucleic Acids?



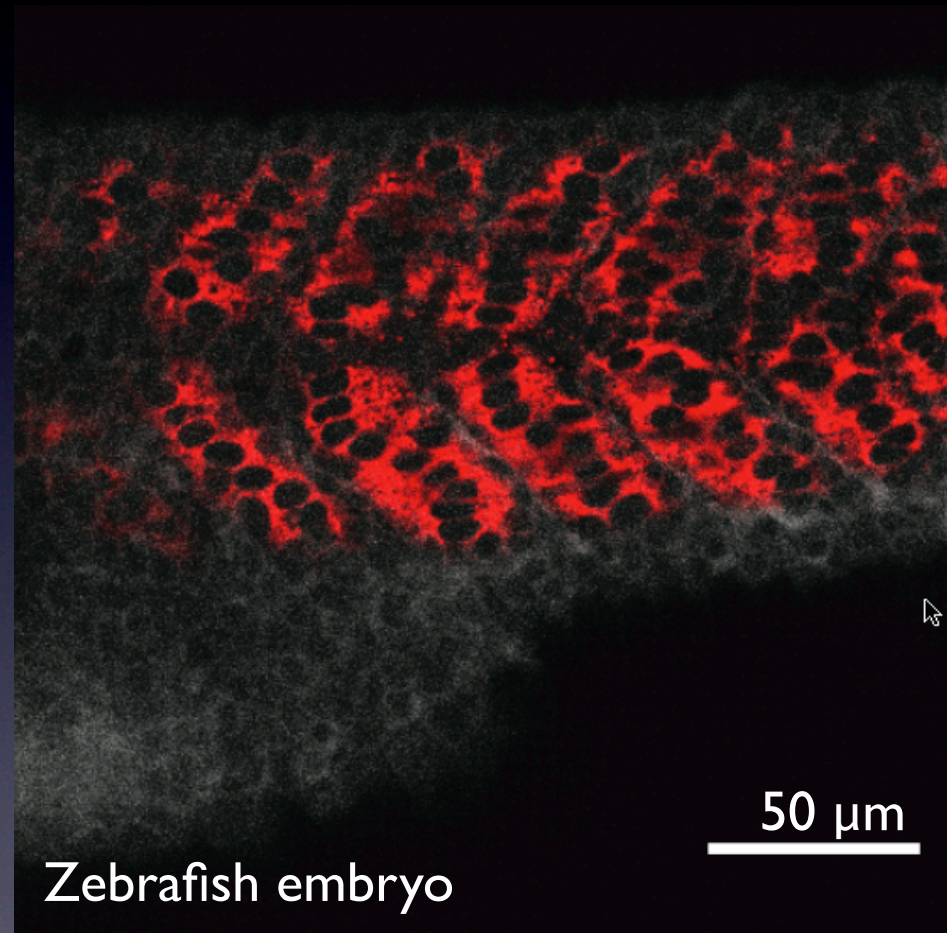
$$\Delta G(\phi, s)$$

# Molecular Instruments: Mapping Biological Circuits within Organisms



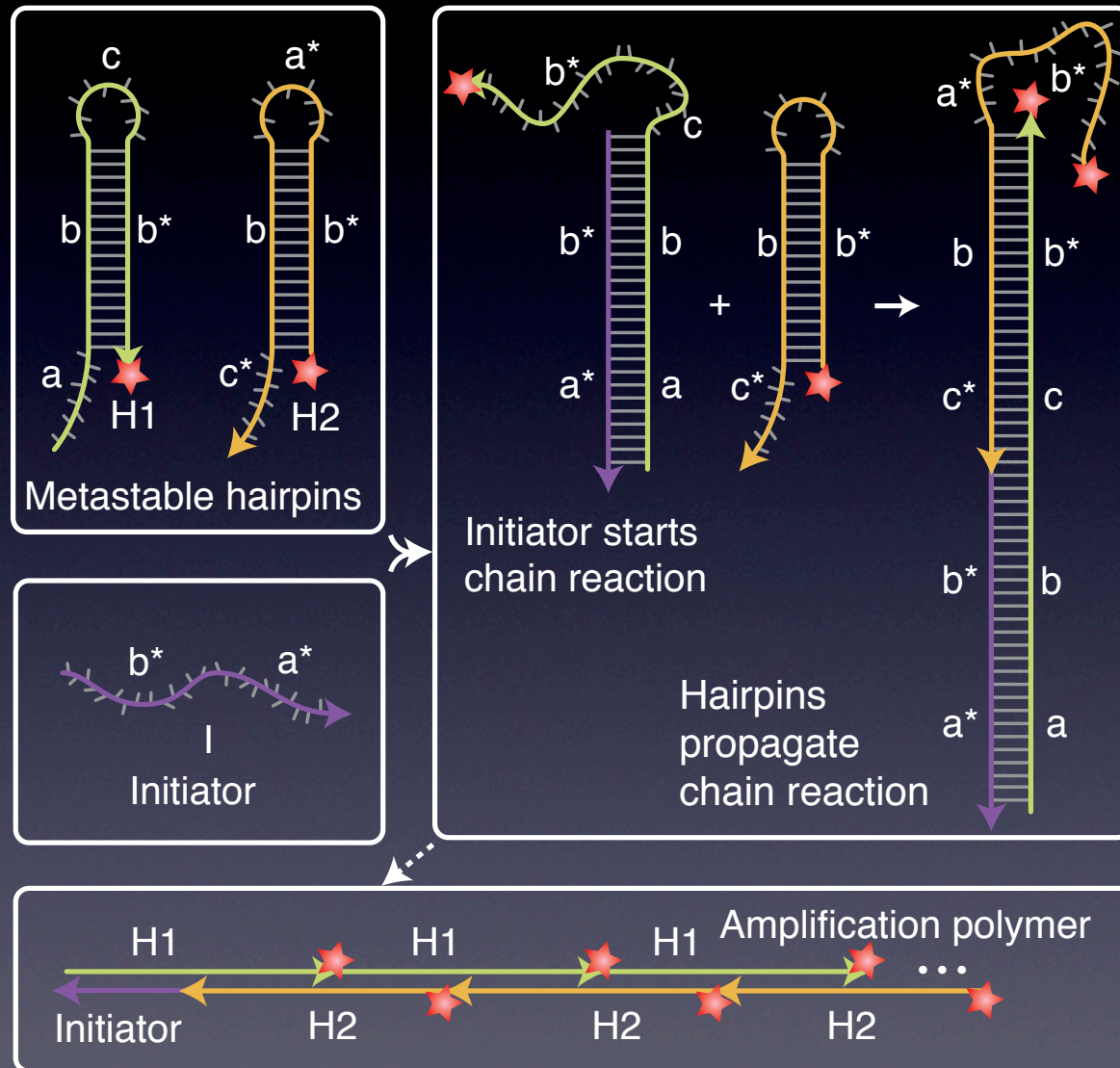
# Mapping mRNA expression within intact vertebrate embryos

**Goal:** accurately map spatial relationships between the regulatory loci of multiple genes

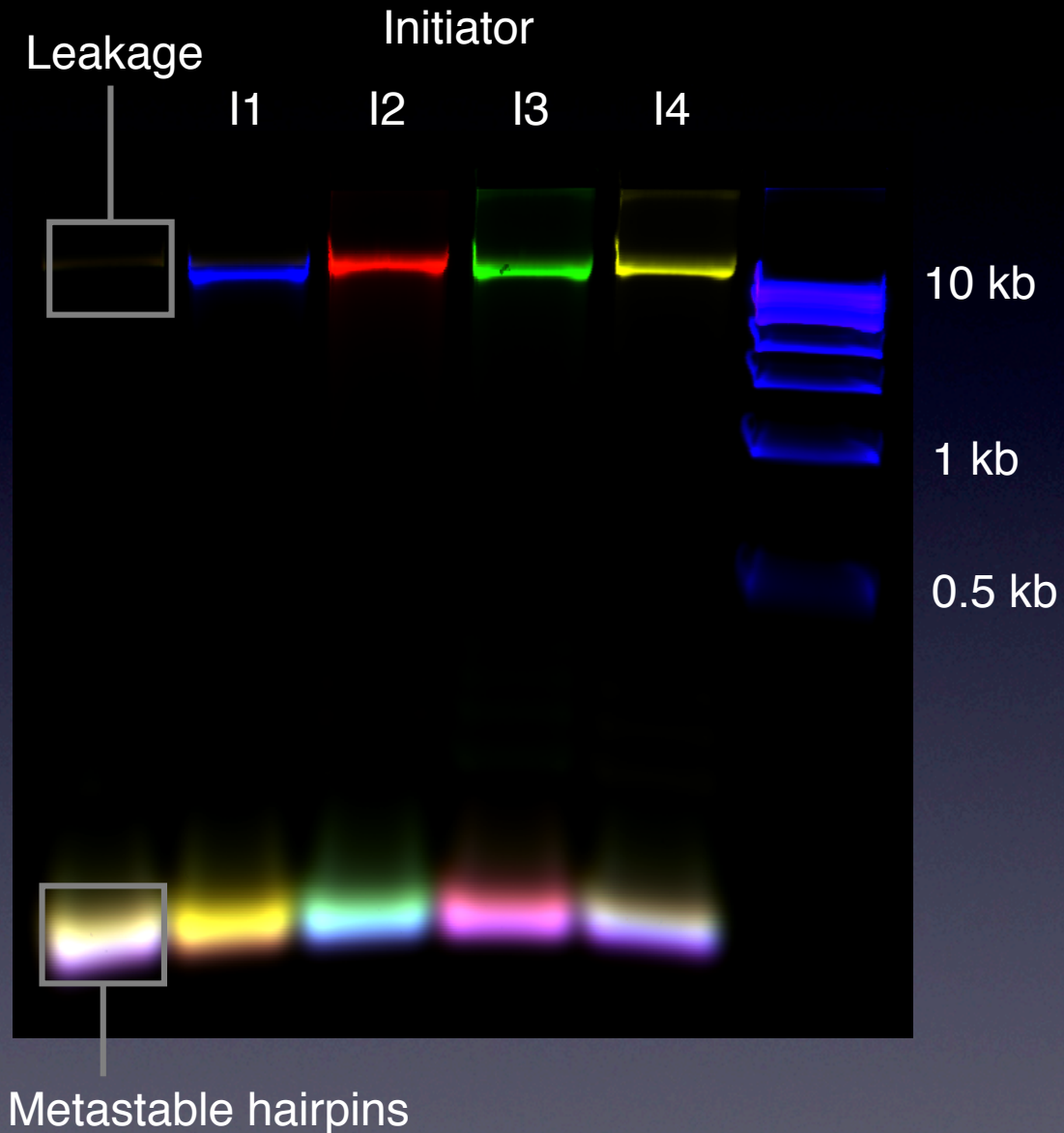


**40-year challenge:** lack of orthogonal in situ amplifiers leads to cumbersome serial multiplexing approaches

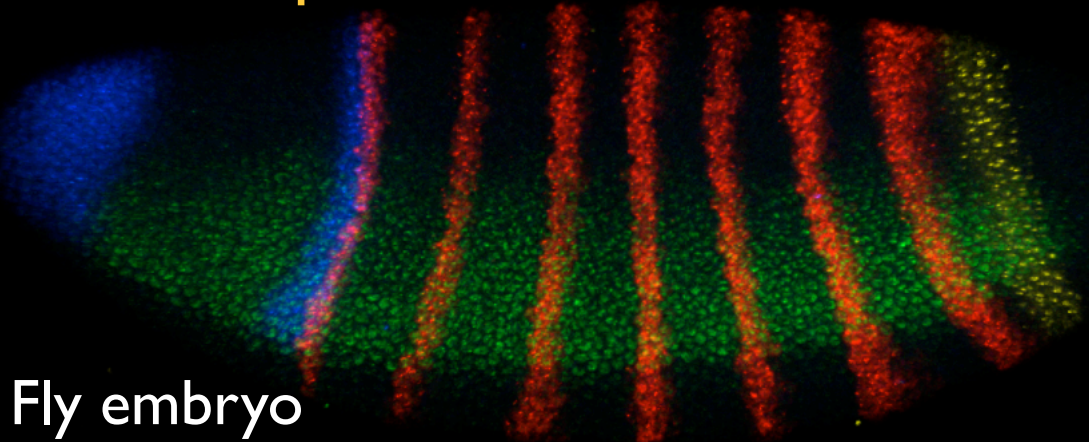
# Conditional Self-Assembly via Hybridization Chain Reaction (HCR)



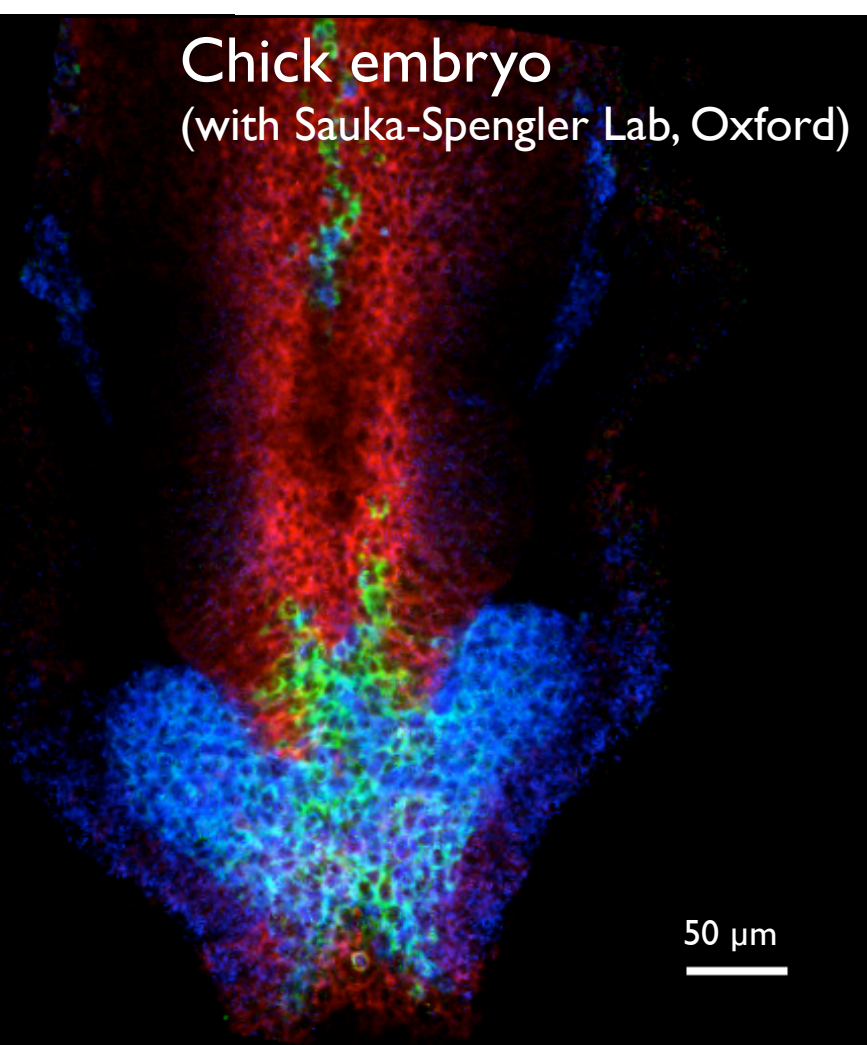
# Multiplexed Signal Amplification using HCR



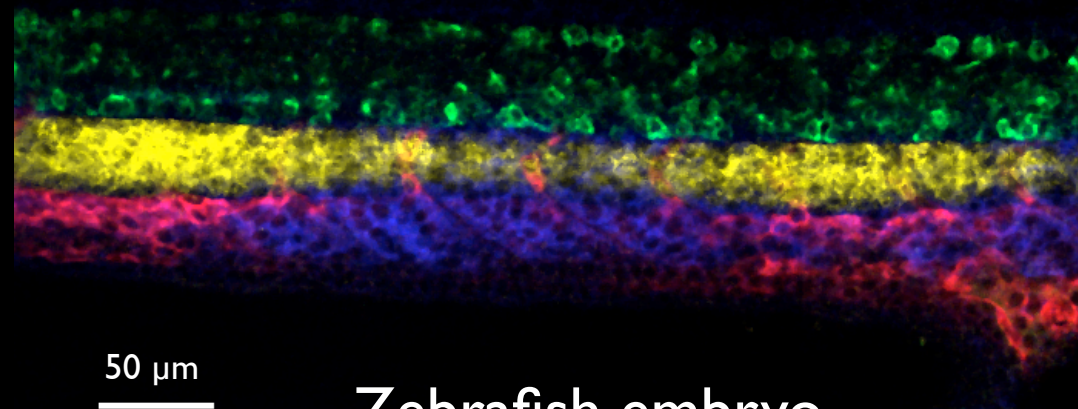
# Multiplexed HCR in situ



Chick embryo  
(with Sauka-Spengler Lab, Oxford)



Mouse embryo  
(with Lansford and Fraser Labs, USC) 200 µm



50 µm  
Zebrafish embryo

mRNAs map to specific cells  
within the developing zebrafish brain

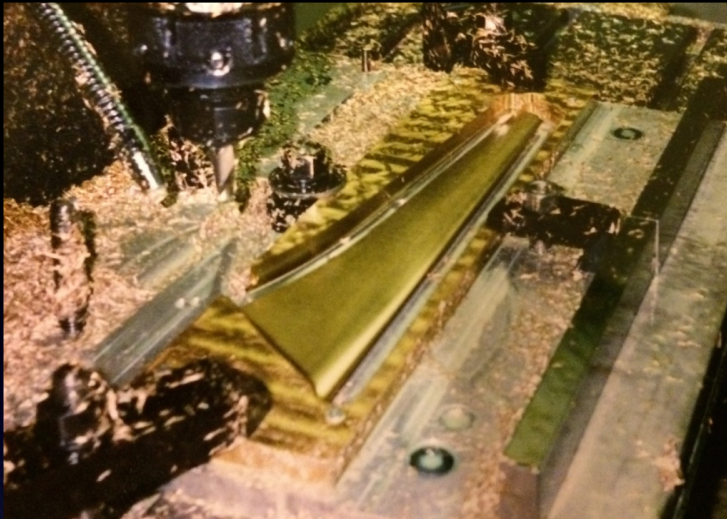
eye



This fluorescence microscopy image shows a cross-section of a developing zebrafish brain. The brain tissue is visible as a light gray, curved structure against a dark background. Two dark, crescent-shaped regions at the top and bottom are labeled 'eye'. The brain tissue contains several clusters of fluorescently labeled mRNAs. On the left side, there are two distinct clusters of red fluorescent signal. In the center, there are two clusters of blue and yellow fluorescent signal. On the right side, there are two clusters of green fluorescent signal. The overall pattern suggests that different mRNAs are localized to specific, non-overlapping regions within the brain.

eye

# Shape

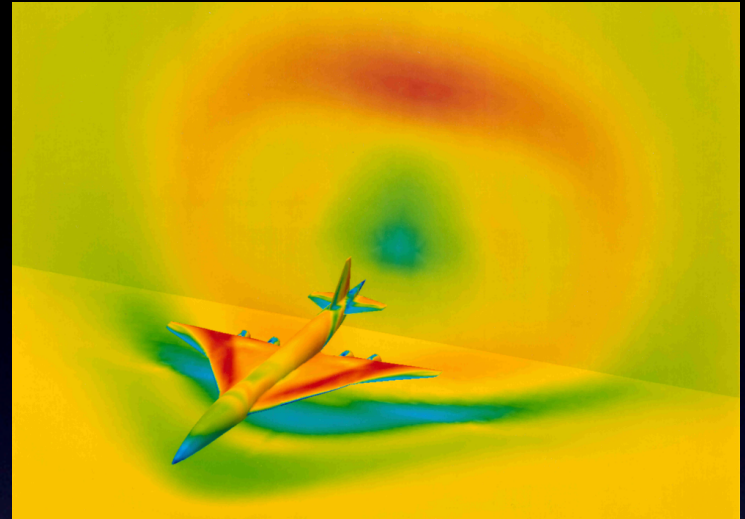


Analysis



Design

# Flow



# Sequence

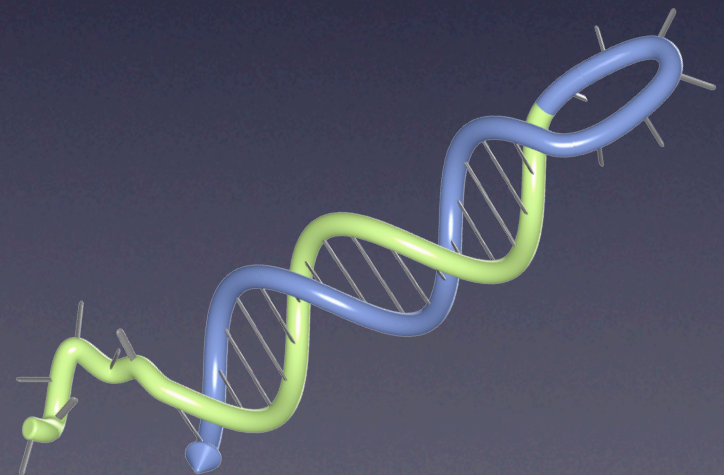
GTGGTACAGGTGAATTTG  
GGTAGGCTAAATTGTCCA  
TAGTTTATGTGTGTGAAT  
GAGGGTGTATGGATGTTT  
CTCAGAGATGGGTTGCAG  
CTGGAAGGGCGTCCATTG  
TGCAAAACATATGAGGGT

Analysis



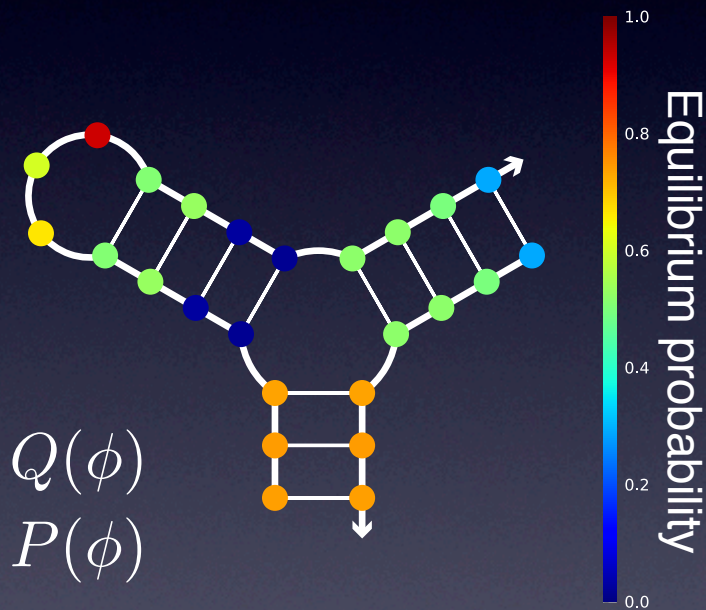
Design

# Shape



# Analysis of a Complex

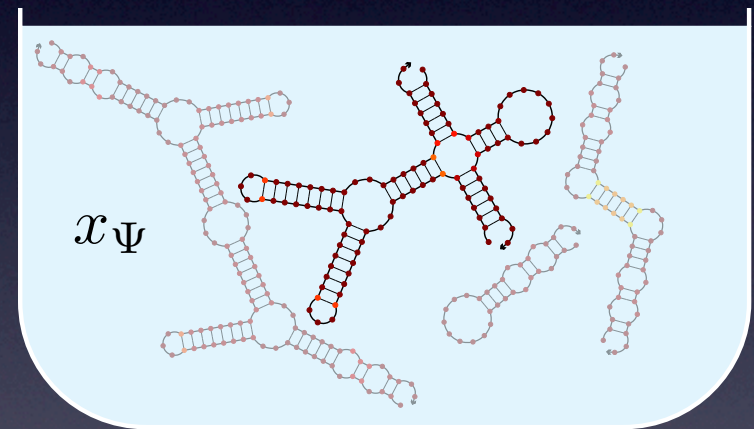
- Arbitrary number of strands
- Dynamic programming
- Graph theory
- Group theory



Complexity:  $\Theta(N^3)$

# Analysis of a Test Tube

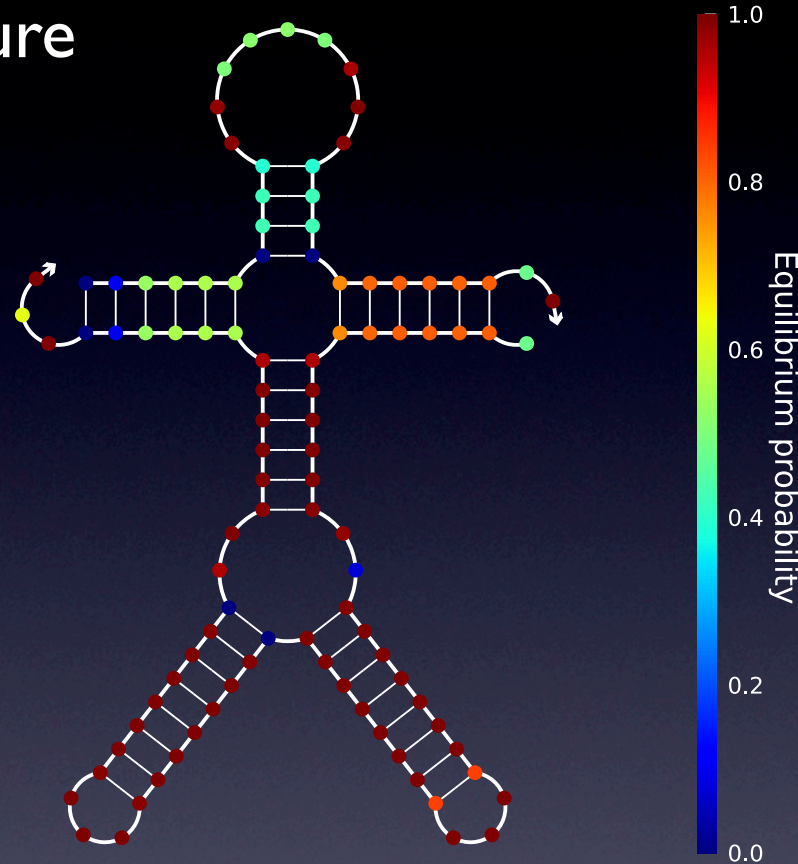
- arbitrary complex species  $\Psi$
- convex programming
- Lagrange duality



Complexity:  $O(|\Psi|N_{\max}^3)$

# Complex Design

Target structure



Objective Function

**Complex ensemble defect:** average number of incorrectly paired nucleotides at equilibrium over the ensemble of the complex

# Cost of Design

**Goal:** Mutate sequence to reduce objective function

**Question:** If each evaluation of the objective function costs

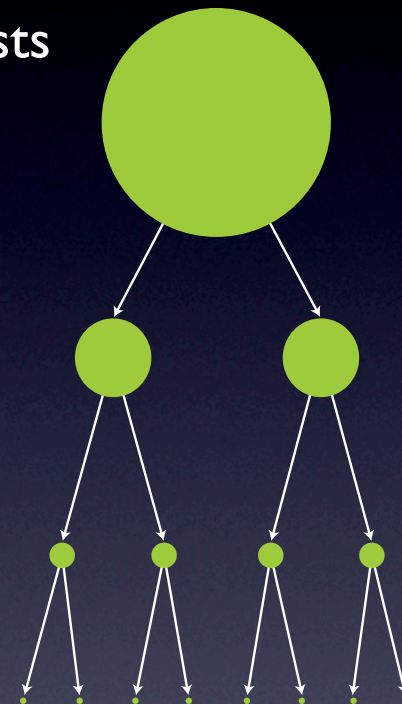
$$c_{\text{eval}}(N) = \Theta(N^3)$$

what is the cost of designing a sequence from scratch?

**Approach:** hierarchical ensemble decomposition

**Optimality bound:**

$$c_{\text{des}}(N) \geq c_{\text{eval}}(N) \left[ 1 + 2\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{4}\right)^3 + 8\left(\frac{1}{8}\right)^3 + \dots \right]$$



# Cost of Design

**Goal:** Mutate sequence to reduce objective function

**Question:** If each evaluation of the objective function costs

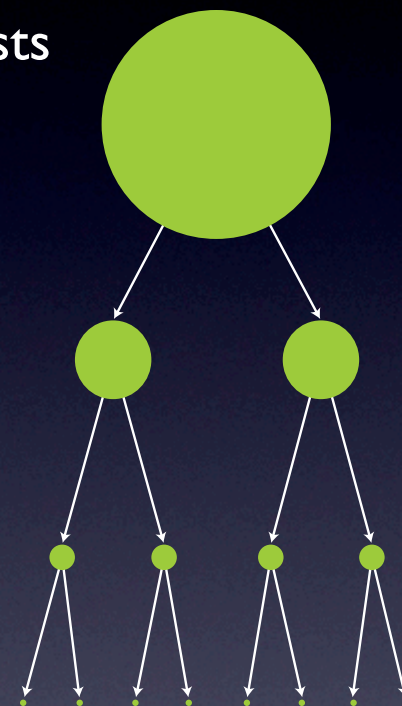
$$c_{\text{eval}}(N) = \Theta(N^3)$$

what is the cost of designing a sequence from scratch?

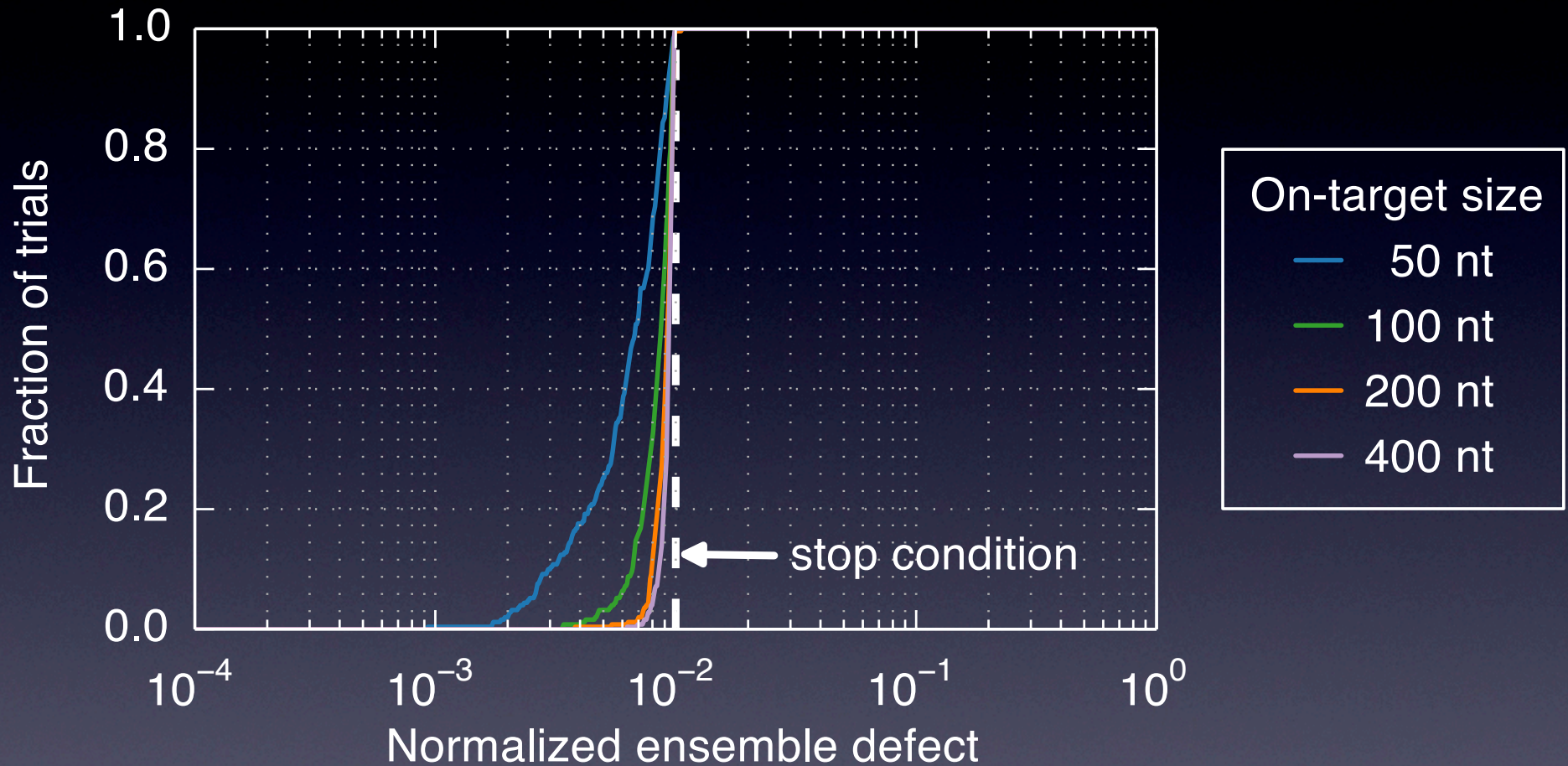
**Approach:** hierarchical ensemble decomposition

**Optimality bound:**

$$c_{\text{des}}(N) \geq \frac{4}{3} c_{\text{eval}}(N)$$

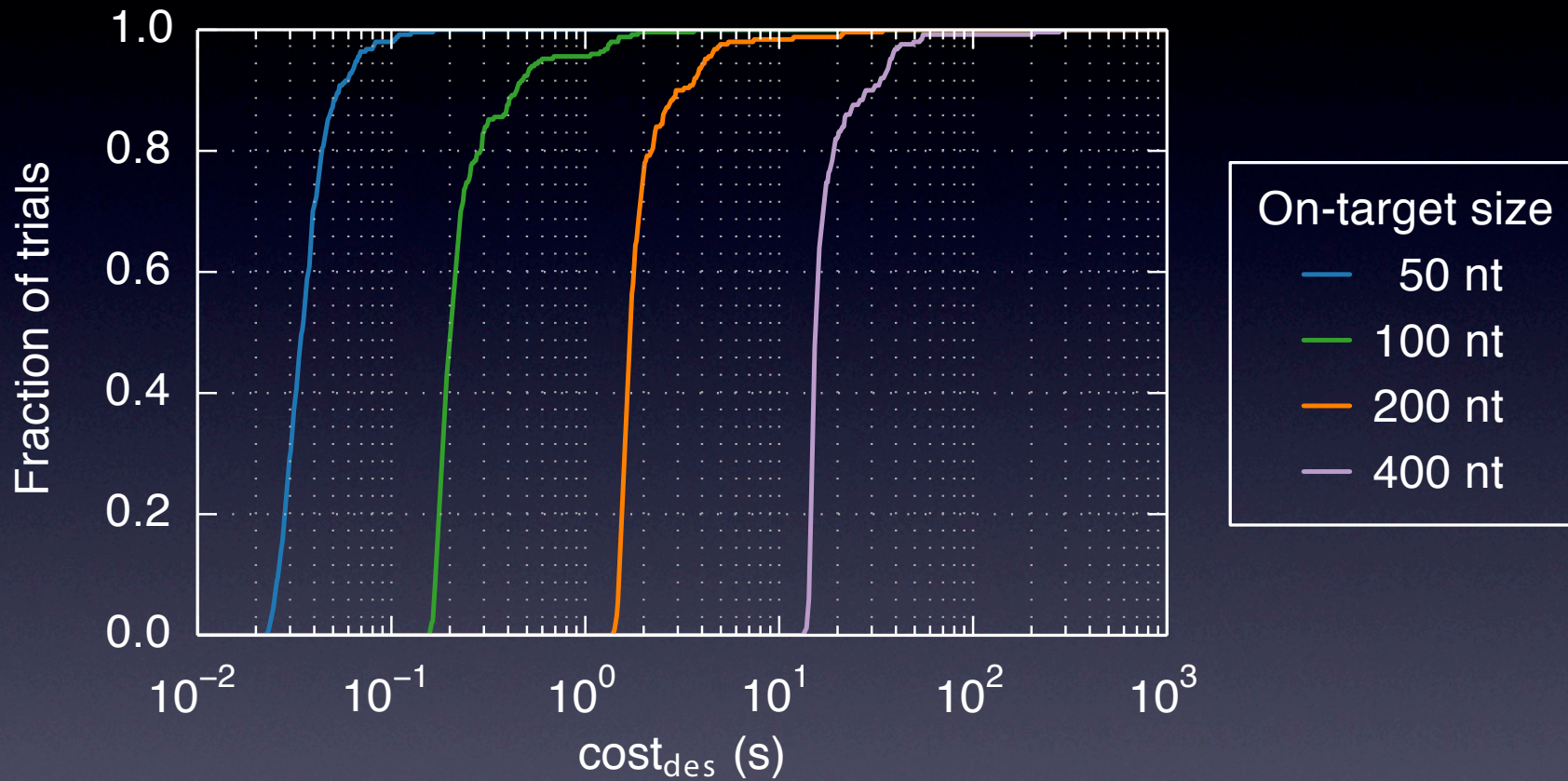


# Design Quality



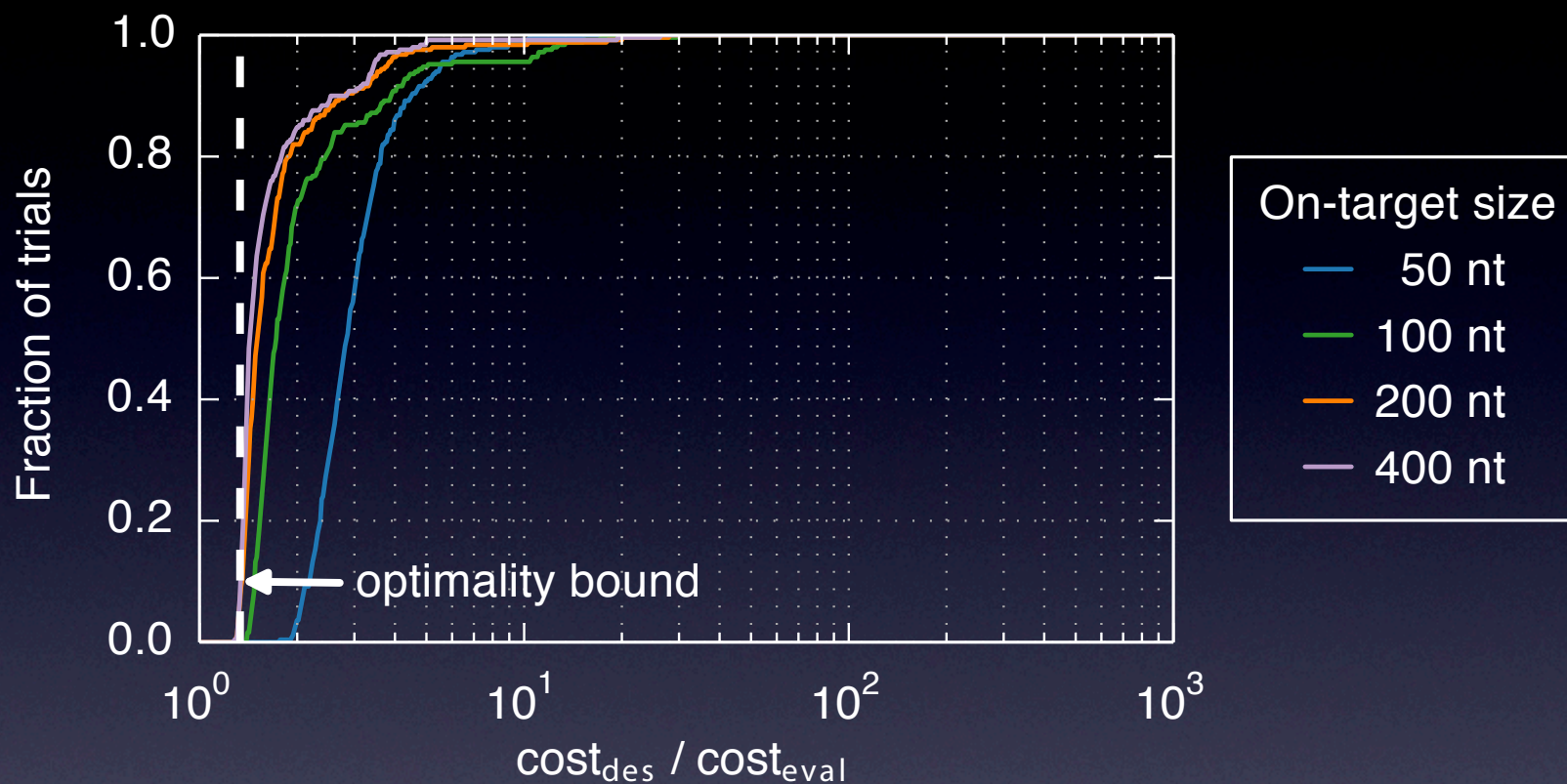
Randomly generated target structures  
Typically <1% of nucleotides incorrectly paired at equilibrium

# Design Cost



Typical design times ranging from fraction of second to seconds

# Relative Design Cost

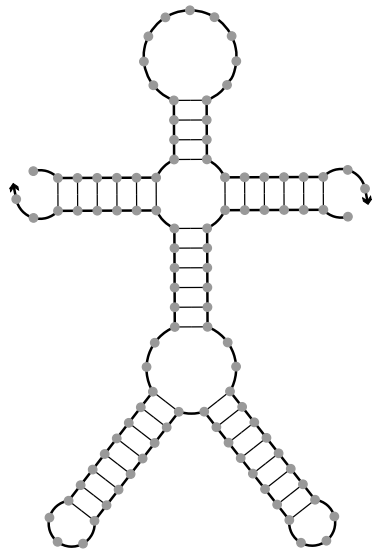


As complex size increases, typical cost of design decreases to 4/3 cost of analysis!

Empirically, algorithm exhibits **asymptotic optimality**

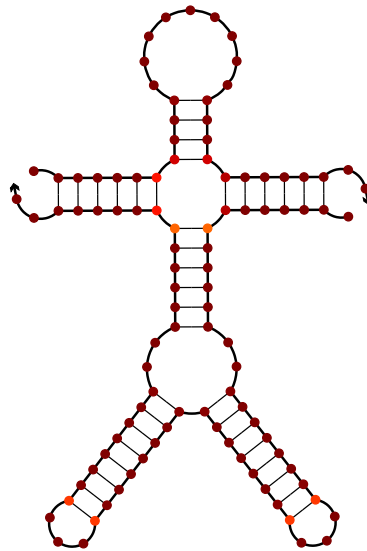
# Complex Design: A Cautionary Tale

Target structure



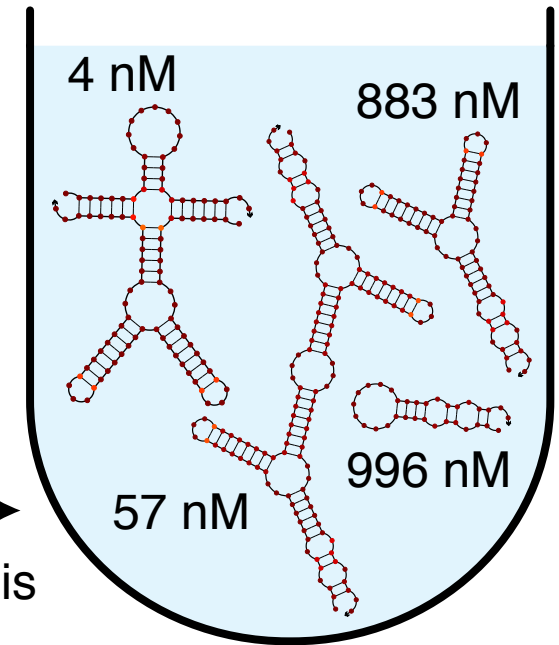
Design

Complex ensemble



Analysis

Test tube ensemble



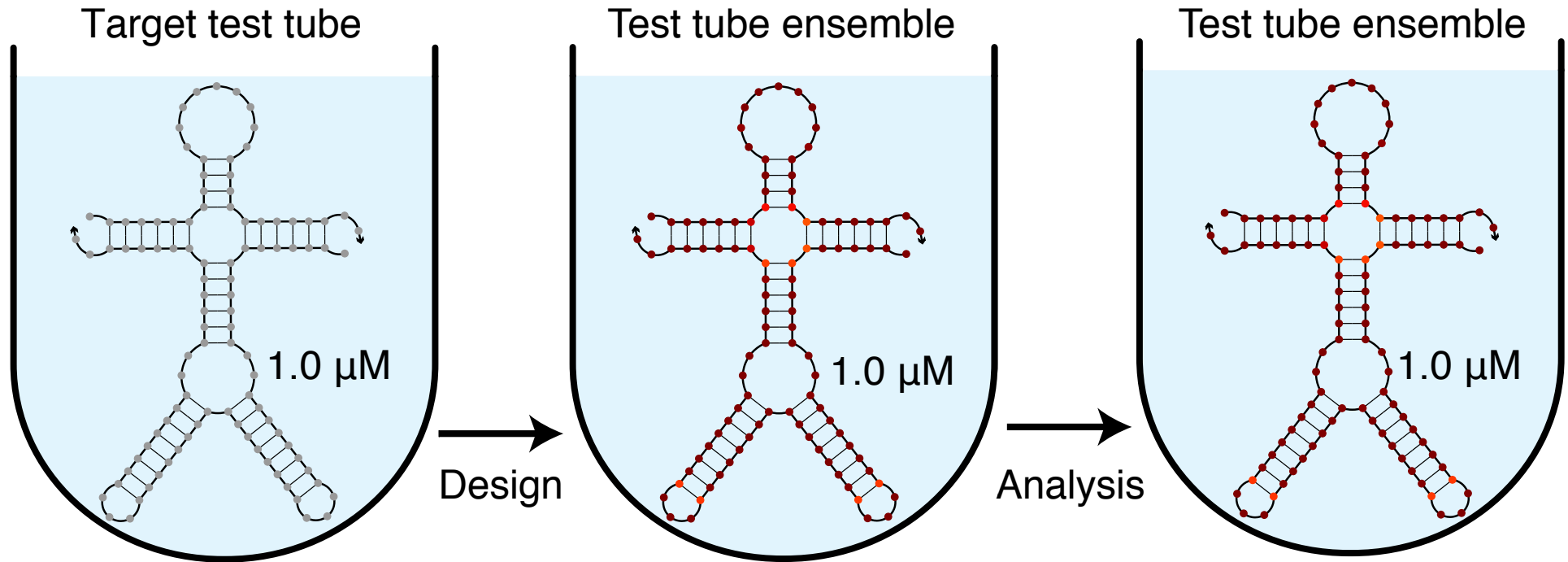
Equilibrium probability

0.0 0.2 0.4 0.6 0.8 1.0

Complex design does not take into consideration:

- concentration of desired complex
- concentration of competing off-target complexes

# Test Tube Design



## On-target complexes:

- target structure
- target concentration

## Off-target complexes:

- vanishing target concentration

Equilibrium probability

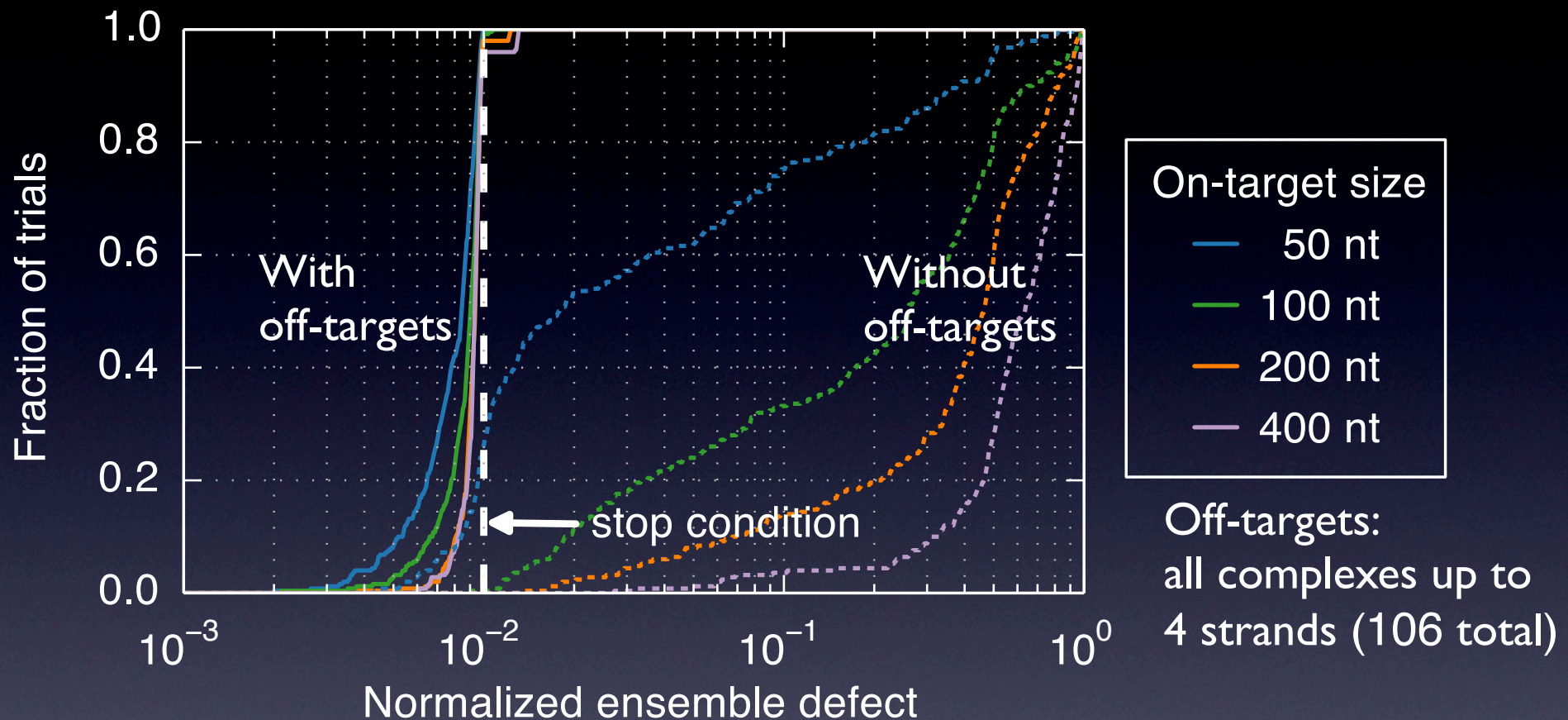
0.0 0.2 0.4 0.6 0.8 1.0



Wolfe and Pierce, *ACS Synth Biol*, 2014

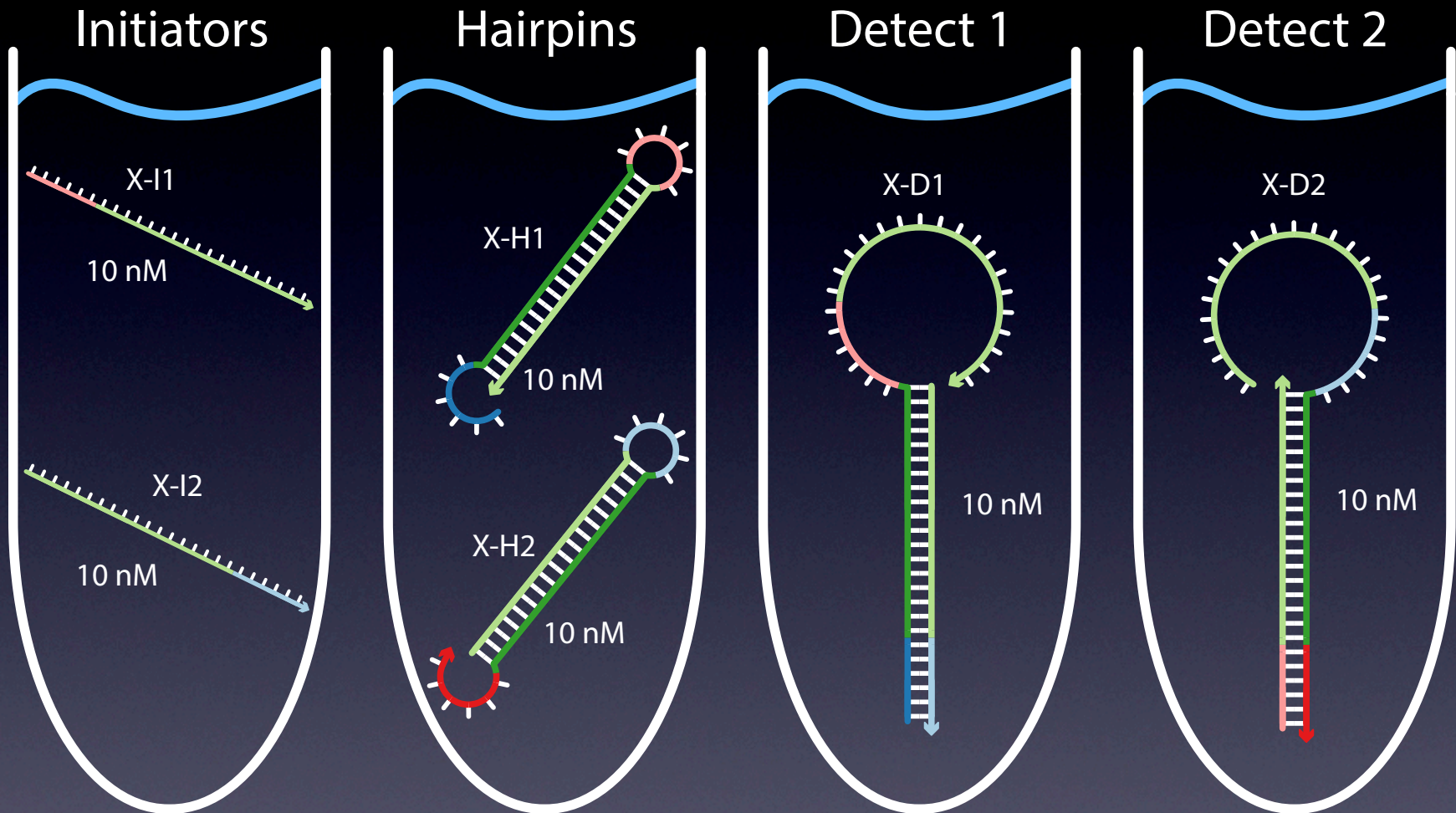
Include 100s to 1000s of off-target complexes  
Typical design cost 2x-10x analysis cost

# Importance of Designing Against Off-Targets



Designing without off-targets in the design ensemble leads to poor sequence quality

# Multistate Test Tube Design



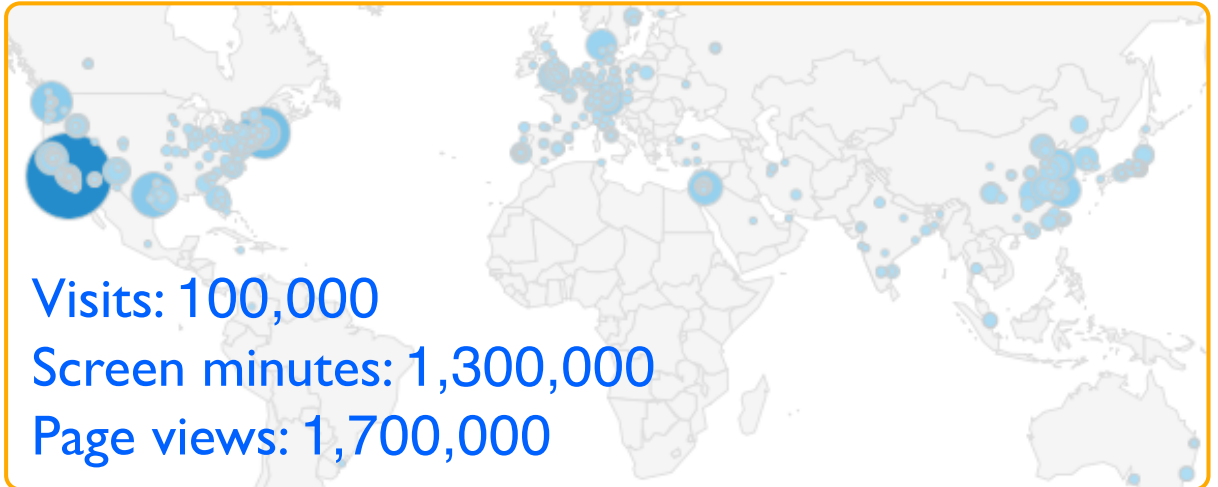
Target test tubes represent reactant, intermediate, and product states

# Non-Profit Academic Resources



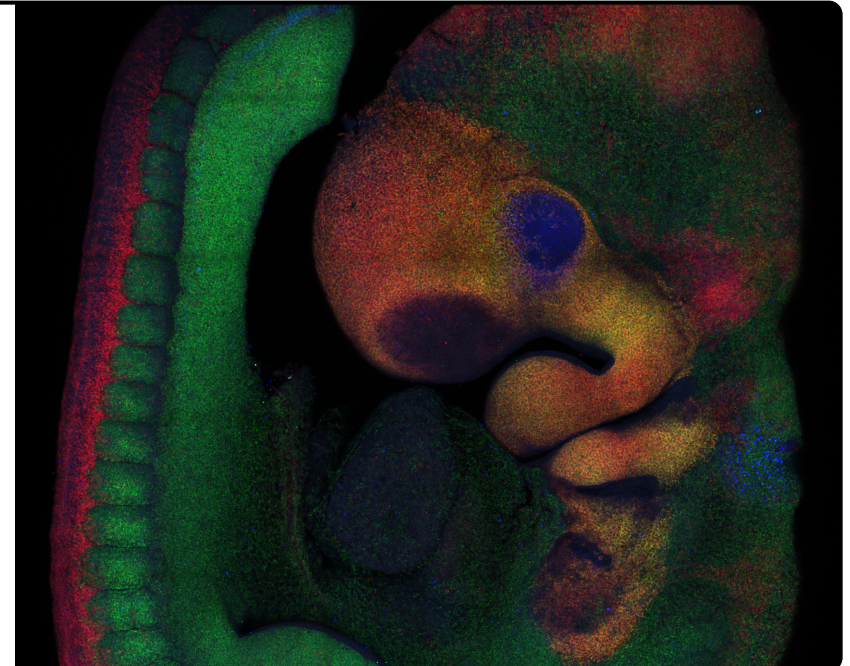
Provide researchers with algorithms for the analysis and design of nucleic acid molecules, devices, and systems

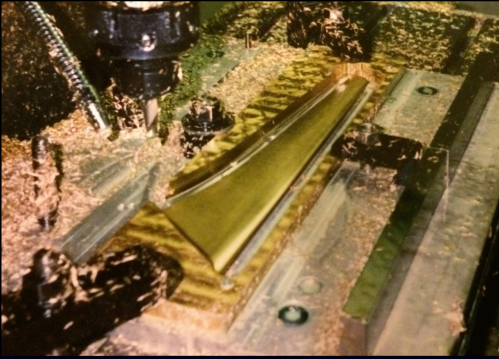
NUPACK usage during the last 3 years



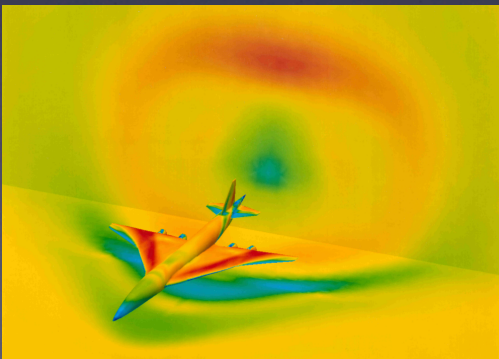
Provide researchers with programmable molecular instruments for readout and regulation

Custom molecular instruments synthesized for over 70 labs





Analysis ↓ ↑ Design



```
GTGGTACAGGTGAATTTGGG
TAGGCTAAATTGTCCATAGT
TTATGTGTGTGAATGAGGGT
GTATGGATGTTTCTCAGAGA
TGGGTTGCAGCTGGAAGGGC
GTCCATTGTGCAAAACATAT
```

Analysis ↓ ↑ Design

